Complete Observables for Canonical General Relativity

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Loops'05

gr-qc/0411013
gr-qc/0507106
PhD Thesis 2005

Motivation and Overview

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Overview

1. One Constraint
2. Arbitrary Many Constraints
3. General Relativity: infinitely many constraints
4. Reducing the Number of Constraints: $\infty \rightarrow$ one
5. Outlook

One Constraint: FRW–Cosmology

Method: C.Rovelli (1990/2002)

Example: FRW–cosmology with massless scalar field

phase space: $x = (a, P_a, \phi, P_\phi)$ constraint: $C = \frac{1}{2}(-\frac{P_a^2}{a} + \frac{P_\phi^2}{a^3})$

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Dirac Observable for arbitrary τ

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$\Rightarrow$ To compute this introduce a new basis of constraints $\tilde{C}_i$

$$\tilde{C}_i = C_k\,(A^{-1})_{ki} \quad \text{with} \quad A_{lm} = \{T_l, C_m\}$$

$$\Rightarrow \quad \{T_k, \tilde{C}_i\} \simeq \delta_{ki} \qquad \Rightarrow \{\tilde{C}_i\} \text{ 'weakly Abelian'}$$

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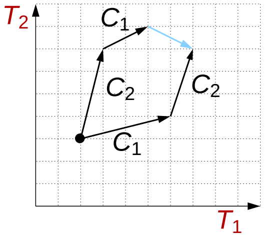

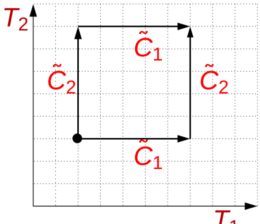

$\tilde{C}_i$ weakly Abelian

Power Series

$$\mathbf{F}^{\mathcal{T}_i}_{[f;\, T_i]}(x) = \sum_{k_i=0}^{\infty} \frac{1}{|k|!} \tilde{C}_1^{k_1} \cdots \tilde{C}_N^{k_N}[f](x)(\tau_1 - T_1(x))^{k_1} \cdots (\tau_N - T_N(x))^{k_N}$$

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acts on f: $\tilde{C}_i[g] := \{g\,,\,\tilde{C}_i\}$

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↑

'distance' of x to $T_i = \tau_i$

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$:= f(x')$ with

x' gauge equivalent to x and

$T_i(x') = \tau_i \quad \forall \quad i$

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Some Results

- complete set of Dirac observables
- Poisson brackets between complete observables

  ($\mathbf{F}_{[\,\cdot\,;\,T_i]}^{\tau_i}$ is a symplecto–morphism onto space of Dirac observables)
- physical Hamiltonians generate evolution wrt clock variables (Kuchař 1972, Rovelli 1990, Thiemann 2004, B.D 2005)
- partial observables invariant under subset of the constraints: "non–perfect clocks"

Complete Observables for GR (B.D. 2005)

- infinitely many constraints: $C_a(\sigma),\ a = 1, 2, 3;\ \ C_\perp(\sigma)$
  $\sigma \in \Sigma$ (spatial hypersurface)

⇝ infinitely many clocks: $T^K(\sigma),\ K = 0, \ldots 3,\ \ \sigma \in \Sigma$

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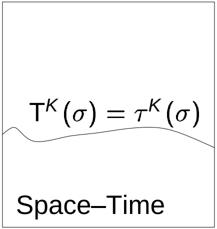

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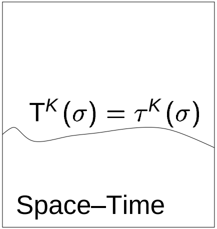

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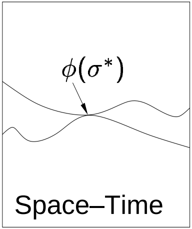
- infinitely many parameters needed to fix embedding
- but: choose f such that it does not depend on all aspects of the embedding

⇝ f invariant under a subset of the constraint: $\rightarrow$ space–time scalar

Reducing the Number of Constraints



- condition for space-time scalar (K. Kuchař 1977):
  $\{C[N, N^a], \phi(\sigma^*)\} = 0$ for $N(\sigma^*) = 0 = N^a(\sigma^*)$

$\leadsto$ $\phi(\sigma^*)$ invariant under almost all constraints

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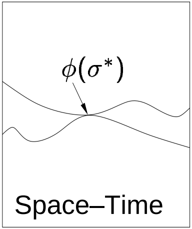

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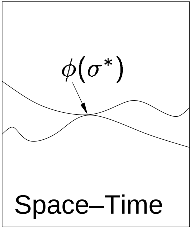

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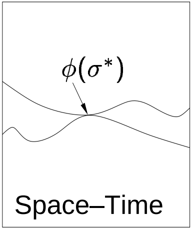

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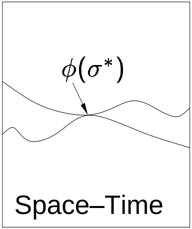

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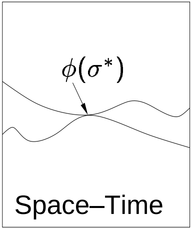

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Need only four constraints for calculation of complete observable

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depends on only four parameters $\tau^K(\sigma^*)$

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⋆ physical Hamiltonian: $\{\mathbf{F}^{\tau^K}_{\gamma^{KL}}, H_M^{\tau^K(\sigma)}\} = \mathbf{F}^{\tau^K}_{\partial_M \gamma^{KL}} = \frac{\partial}{\partial \tau_M} \mathbf{F}^{\tau^K}_{\gamma^{KL}}$

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Need only one constraint for calculation of complete observable!

choose another scalar $T^0(\tau^C)$: $\tilde{C}(\tau^C) = [\{T^0(\tau^C), C_\perp[1]\}]^{-1} C_\perp(\tau^C)$

$$\mathbf{F}^{\tau^0}_{\gamma^{KL}(\tau^C)} = \sum_k \frac{1}{k!} \tilde{C}[1]^k \, [\gamma^{KL}(\tau^C)](\tau^0 - T^0(\tau^C))^k$$

Conclusions and Outlook

- partial and complete observables: method to systematically investigate observables and their algebra
- can express observables of the covariant formalism in the canonical formalism: compare quantizations
- reduction to one constraint $\rightsquigarrow$ approximation scheme
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⋆ good clock variables? $\rightsquigarrow$ physical Hamiltonian (positive, τ–independent)

⋆ how to deal with bad clock variables?

⋆ quantization: different clock variables (with T. Thiemann)

Appendix

$$C_\perp(\tau^C) = \frac{1}{\kappa} g^{-\frac{1}{2}} (p_{AB} p^{AB} - \frac{1}{2} p^2)(\tau^C) - \frac{1}{\kappa} g^{\frac{1}{2}} R(\tau^C) + \frac{1}{2\alpha_0} g^{-\frac{1}{2}} (\Pi_0)^2 (\tau^C) + \frac{1}{2\alpha_0} g^{\frac{1}{2}} g^{AB} T^0_{,A} T^0_{,B} (\tau^C) + \frac{1}{2} g^{-\frac{1}{2}} (Y) \sum_{A=1}^{3} \frac{1}{\alpha_A} (\Pi_A)^2 (\tau^C) + \frac{1}{2} g^{\frac{1}{2}} (Y) \sum_{A=1}^{3} \frac{1}{\alpha_A} g^{AA} (\tau^C) + g^{\frac{1}{2}} \sum_{K=0}^{3} \frac{1}{\alpha_K} V_{(K)}(T^0(\tau^C), \tau^C)$$