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Singularities in
Loop Quantum Gravity:

A Status Report

Max Planck Institut
für
Gravitationsphysik,
Golm near Potsdam,
Germany

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LOOPS'05 @ AEI

Golm, 14/10/2005

Outline

- Singularities in GR: A General Concept

  → Cosmological Models-FRW

- LQG-Elements ↔ LQC-Elements

  → General Structures → Results of Symmetry Reduction

- Calculation in Full LQG

  → Inverse Scale Factor at gauge invariant 3-vertex ($SU(2)$)

  → CS-Expectation-Value at M-vertex ($U(1)^3$)

- Current Investigations

Motivation: Singularity Theorems

- Very general statements, robust wrt. possible symmetry reductions

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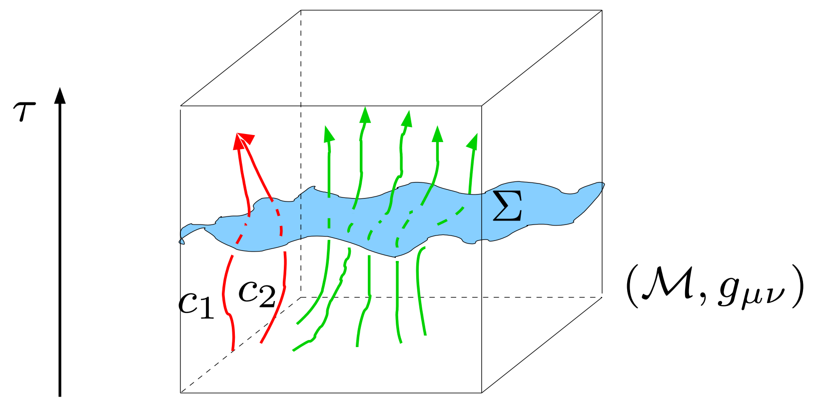
- Situation



provide criteria when $(\mathcal{M}, g_{\mu\nu})$ contains at least 1 incomplete timelike or 0-geodesic (within finite proper time τ_c the geodesic can't be extended)

$\Rightarrow$ Classification:

(i) Scalar from ${R_{\alpha\beta\gamma}}^{\delta}$ blows up

(ii) Component of ${R_{\alpha\beta\gamma}}^{\delta}$ in parallelly propagated tetrad blows up

(iii) No such scalar or component blows up

Cosmology: Simplest Example FRW

- Demand: Spatial homogeneity, isotropy

  $\rightarrow$that gives a natural $3+1$ split of $(\mathcal{M}, g_{\mu\nu})$ and fixes the form of metric and line element

$$ds^2 = -d\tau^2 + a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\Theta^2 + \sin^2\Theta d\phi^2\right)\right]$$

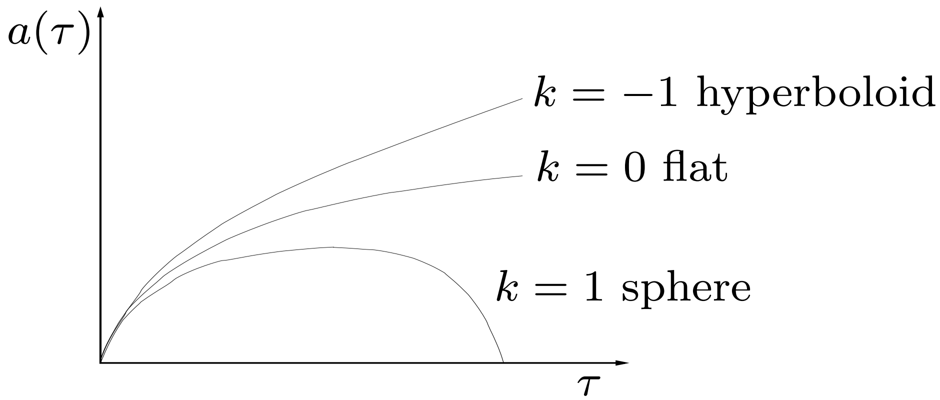
- Friedmann Equations

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\frac{k}{a^2}$$

$$3\frac{\ddot{a}}{a} = -4\pi G_N \left(\rho + 3P\right)$$

$\rho \ldots$ matter density

$P \ldots$ matter pressure



origin: contraction of space to 0, $\rho, R \rightarrow \infty$

$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ for P=0 (dust)

Cosmology: Scalar field (Inflaton)

- Hamiltonian for FRW with homogeneous, isotropic scalar field ϕ

$$H_{kin}(N) = \int_\sigma d^3x\ N\left[\frac{\pi^2}{2a^3} + a^3 V(\phi)\right]$$

  - $\lim_{\tau\to 0} a(\tau)$: singularity prominent in the first term

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- (General) Hamiltonian for scalar field ϕ in 3+1 split

$$H_{kin}(N) = \int_\sigma d^3x\ N\left[\underbrace{\frac{\pi^2}{\sqrt{\det q}}}_{\propto a^{-3}} + \underbrace{\sqrt{\det q}}_{\propto a^3}\left(q^{ab}\phi_{,a}\phi_{,b} + V(\phi)\right)\right]$$

  - homogeneous $\rightsquigarrow$ derivative term drops out
  - $\lim_{\tau\to 0} a(\tau)$: singularity prominent in the first term

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Can be promoted to bounded operator in LQC. What about LQG?

LQG⇔LQC: Comparison

	LQG	LQC
Similar	Quantization methods Poisson-Identity	
Different	QFT $T_{\gamma \vec{j} \vec{I}}$ (non abelean, arbitrary γ) discrete volume spectrum highly degenerated 0-eigenvalues $\hat{C}_{LQG}$ modifies graph $\hat{C}_{LQG}$ bounded ?	minisuperspace T_r (effectively abelean, fixed γ) continuous volume spectrum single 0-eigenvalue $\hat{C}_{LQC}$ leaves graph invariant $\hat{C}_{LQC}$ bounded on 0-volume states (contrary to classical behaviour!)

$$\hat{C}_{LQC} = -\alpha q^2 \sqrt{|p|} + b \frac{\pi^2}{\sqrt{|p|}^3} + \sqrt{|p|}^3 V(\phi)$$

$$\hat{C}_{LQG} = \frac{\left[\epsilon_{jkl}\left[(1+\beta^2)\epsilon_{jmn} K_a^m K_b^n\right] E_k^a E_l^b\right]}{\sqrt{|\det E|}} + \frac{1}{2}\left[\frac{\pi^2}{\sqrt{|\det E|}} + \frac{E_j^a E_j^b}{\sqrt{|\det E|}} \phi_{,a} \phi_{,b} + \sqrt{|\det E|} V(\phi)\right]$$

Need to understand the relations between LQG and LQC!

LQG$\Leftrightarrow$LQC: Questions

LQG⇔LQC: Questions

- Validity of Assumptions:

  ? Under which circumstances a given symmetry reduction can be applied without fading away characteristic properties of the theory?

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- 'Backreaction'

  ? Is it possible that within the reduced dynamics a regime is reached in which the original assumptions justifying the symmetry reduction are violated?

LQG⇔LQC: Questions

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  ? Under which circumstances a given symmetry reduction can be applied without fading away characteristic properties of the theory?

- 'Backreaction'

  ? Is it possible that within the reduced dynamics a regime is reached in which the original assumptions justifying the symmetry reduction are violated?

- 'Problem or Feature'

  ? When applying a symmetry reduction life simplifies a lot. E.g. the valence of vertices is fixed, the theory becomes effectively abelian, as a result operators as (inverse) volume are simplified. Do certain properties result from symmetry reduction or do they still mirror physical properties of the particular model?

Kinematical Term of Scalar Field in Hamilton Constraint Operator

- $C^{scalar}_{kin}(N) = \int_\sigma d^3x\ N\left[\frac{\pi^2}{\sqrt{\det q}}\right] = \int_\sigma d^3x\ N\pi^2\left[\frac{\left[\det(e)\right]^2}{\left[\det q\right]^{\frac{3}{2}}}\right]$

  $\rightarrow$ singularity prominent in this term

- According operator in Full LQG

$$\hat{C}^{scalar}_{kin}(N)\ \ T_{\gamma\vec{j}\vec{I}\vec{n}} = \sum_{v\in V(\gamma)} N(v)\quad \hat{H}^{(grav)}_{kin}\quad T_{\gamma\vec{j}I}\otimes \hat{H}^{(matter)}_{kin}\ \ T_{v(\gamma)\vec{n}}$$
$$= \sum_{v\in V(\gamma)} N(v)\ \left[\widehat{\frac{1}{\sqrt{\det q}}}(v)\right] T_{\gamma\vec{j}I}\otimes\quad \hat{\pi}^2\quad T_{v(\gamma)\vec{n}}$$

- where

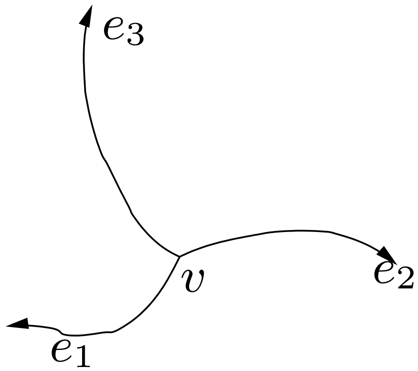
$$\hat{H}^{(grav)}_{kin,v} = \left[\frac{1}{\hbar^3\kappa^3}\frac{1}{E(v)}\sum_{IJK}\epsilon(IJK)\ \epsilon^{IJK}\ \ \epsilon_{ijk}\ {}^{(\frac{1}{2})}\hat{e}^i_I(v)\ {}^{(\frac{1}{2})}\hat{e}^j_J(v)\ {}^{(\frac{1}{2})}\hat{e}^k_K(v)\right]^2$$

$$\hat{H}^{(matter)}_{kin,v} = \left[\hbar\kappa X(v)\right]^2$$

$${}^{(r)}\hat{e}^k_K(v)\ \ :=\ \ \mathrm{Tr}\left[\ \tau_k\ h_K\ \left[h^{-1}_K\ ,\ \hat{V}^r_v\right]\ \right]$$

Action on 3-valent (Gauge Invariant) States: Setup

- 3-Valent Vertex v



- Simplest Example for zero Volume Eigenstate

$$\hat{V}_v \,|\, 0 \,\rangle = 0 \cdot |\, 0 \,\rangle$$

- Want

$$\|\hat{H}^{(grav)}_{kin,v}\| := \left| \langle\, 0 \,|\, \hat{H}^{(grav)}_{kin,v} \,|\, 0 \,\rangle \right|$$

Action on 3-valent (Gauge Invariant) States: Result

$$
\begin{aligned}
\| \; \hat{H}^{(grav)}_{kin,v} \; \| = const \cdot \frac{32}{9\,(1+2\,j_1)^2\,(1+2\,j_2)^2\,(1+2\,j_3)^2} \\
\times \; \Big[108\, A_1\, A_2\, A_3 \\
-3\left(2\,(-1)^{2\,j_1} + (-1)^{2\,j_3}\right)^2 A_2\,(A_1 - A_2 + A_3)^2 \\
-3\left(2\,(-1)^{2\,j_2} + (-1)^{2\,j_3}\right)^2 A_1\,(-A_1 + A_2 + A_3)^2 \\
-3\left(1 + 2\,(-1)^{2\,(j_1+j_2)}\right)^2 A_3\,(A_1 + A_2 - A_3)^2 \\
- \; \left(1 + 2\,(-1)^{2\,(j_1+j_2)}\right)\left(2\,(-1)^{2\,j_1} + (-1)^{2\,j_3}\right)\left(2\,(-1)^{2\,j_2} + (-1)^{2\,j_3}\right) \\
\times\,(-A_1 + A_2 + A_3)\,(A_1 - A_2 + A_3)\,(A_1 + A_2 - A_3)\Big] \\
\times \; \left(V_{1A}{}^{\frac{1}{4}} - V_{1B}{}^{\frac{1}{4}}\right)^2 \left(V_{2A}{}^{\frac{1}{4}} - V_{2B}{}^{\frac{1}{4}}\right)^2 \left(V_{3A}{}^{\frac{1}{4}} - V_{3B}{}^{\frac{1}{4}}\right)^2
\end{aligned}
$$

Action on 3-valent (Gauge Invariant) States: Result

$$
\| \hat{H}^{(grav)}_{kin,v} \| = const \cdot \frac{32}{9\,(1+2\,j_1)^2\,(1+2\,j_2)^2\,(1+2\,j_3)^2}
$$

$$
\begin{aligned}
\times\ \big[& 108\,A_1\,A_2\,A_3 \\
& -3\left(2\,(-1)^{2\,j_1} + (-1)^{2\,j_3}\right)^2 A_2\,(A_1 - A_2 + A_3)^2 \\
& -3\left(2\,(-1)^{2\,j_2} + (-1)^{2\,j_3}\right)^2 A_1\,(-A_1 + A_2 + A_3)^2 \\
& -3\left(1 + 2\,(-1)^{2\,(j_1+j_2)}\right)^2 A_3\,(A_1 + A_2 - A_3)^2 \\
& - \left(1 + 2\,(-1)^{2\,(j_1+j_2)}\right)\left(2\,(-1)^{2\,j_1} + (-1)^{2\,j_3}\right)\left(2\,(-1)^{2\,j_2} + (-1)^{2\,j_3}\right) \\
& \qquad \times (-A_1 + A_2 + A_3)\,(A_1 - A_2 + A_3)\,(A_1 + A_2 - A_3) \big]
\end{aligned}
$$

$$
\times\ \left({V_{1A}}^{\frac{1}{4}} - {V_{1B}}^{\frac{1}{4}}\right)^2 \left({V_{2A}}^{\frac{1}{4}} - {V_{2B}}^{\frac{1}{4}}\right)^2 \left({V_{3A}}^{\frac{1}{4}} - {V_{3B}}^{\frac{1}{4}}\right)^2
$$

NOT bounded!

Action on 3-valent (Gauge Invariant) States: Result

Here

$$V_{1A} = \left[(-j_1 + j_2 + j_3 + 1)(j_1 - j_2 + j_3)(j_1 + j_2 - j_3)(j_1 + j_2 + j_3 + 1)\right]^{\frac{1}{2}}$$

$$V_{1B} = \left[(-j_1 + j_2 + j_3)(j_1 - j_2 + j_3 + 1)(j_1 + j_2 - j_3 + 1)(j_1 + j_2 + j_3 + 2)\right]^{\frac{1}{2}}$$

$$V_{2A} = \left[(-j_1 + j_2 + j_3)(j_1 - j_2 + j_3 + 1)(j_1 + j_2 - j_3)(j_1 + j_2 + j_3 + 1)\right]^{\frac{1}{2}}$$

$$V_{2B} = \left[(-j_1 + j_2 + j_3)(j_1 - j_2 + j_3)(j_1 + j_2 - j_3 + 1)(j_1 + j_2 + j_3 + 2)\right]^{\frac{1}{2}}$$

$$V_{3A} = \left[(-j_1 + j_2 + j_3)(j_1 - j_2 + j_3)(j_1 + j_2 - j_3 + 1)(j_1 + j_2 + j_3 + 1)\right]^{\frac{1}{2}}$$

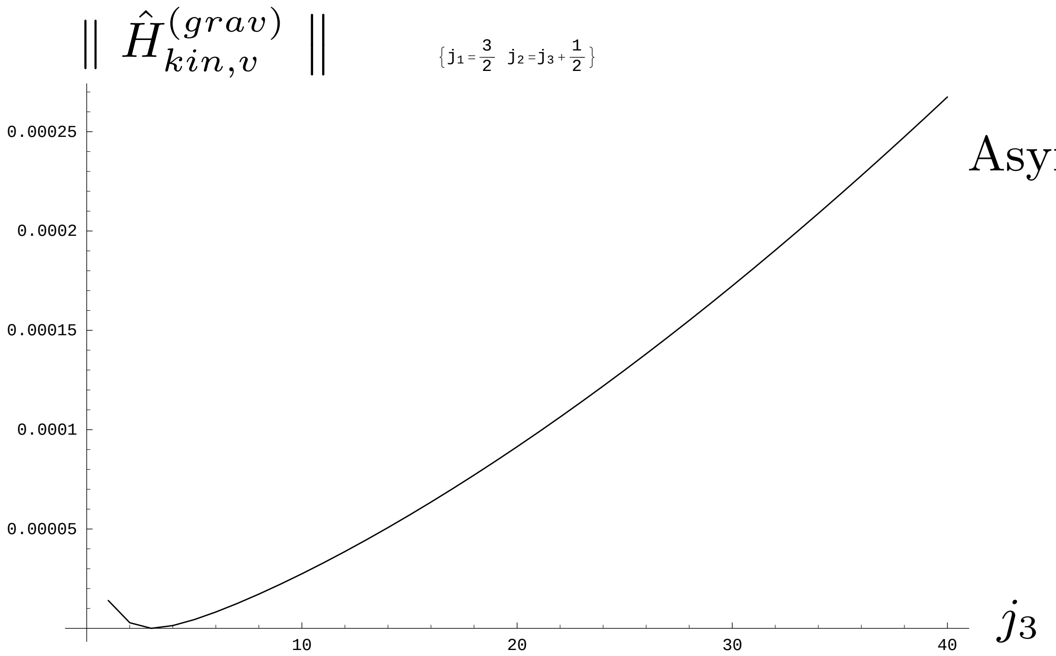
$$V_{3B} = \left[(-j_1 + j_2 + j_3 + 1)(j_1 - j_2 + j_3 + 1)(j_1 + j_2 - j_3)(j_1 + j_2 + j_3 + 2)\right]^{\frac{1}{2}}$$

Moreover $\boxed{A_K = j_K(j_K + 1)}$, $const = \dfrac{6^3 \left|Z\right|^{\frac{3}{2}}}{(\ell_P)^3}$

Plot of the Result: Increasing

If we set $j_1 = \frac{3}{2}$, $j_2 = j_3 + \frac{1}{2}$ where $j_3 \in \mathbb{N}$, $1 \leq j_3 \leq 40$ we get:

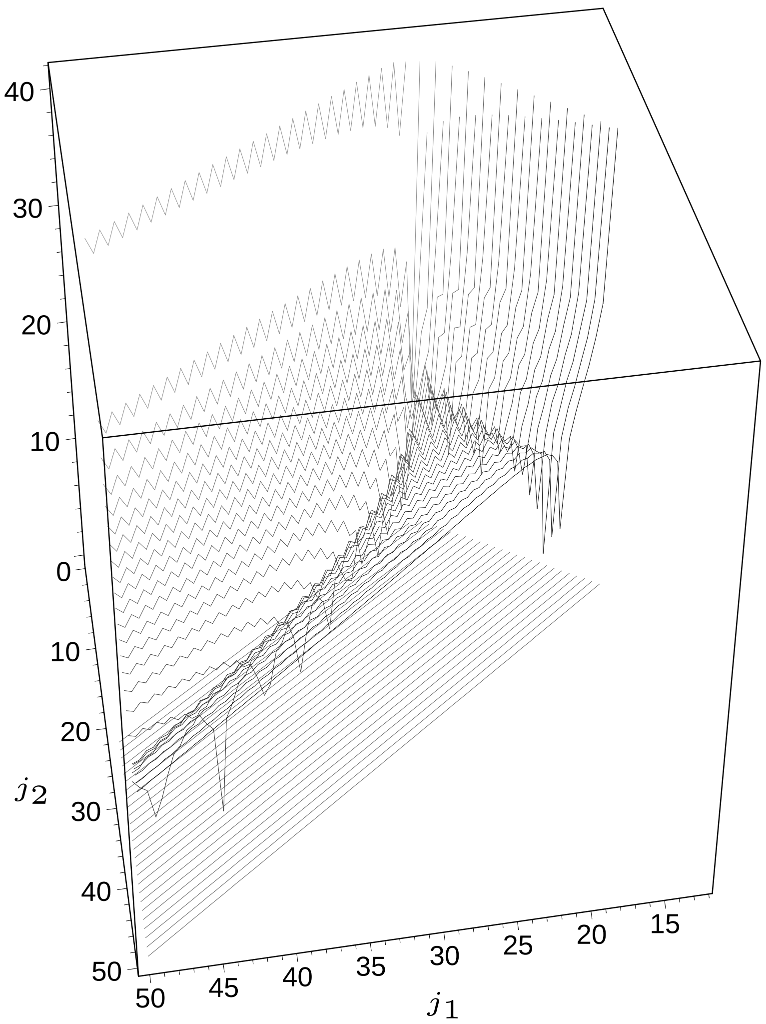
$$\| \hat{H}^{(grav)}_{kin,v} \| \propto \frac{1}{3\,(1+j_3)\,(1+2\,j_3)^2} \times 4\,j_3\,\left(-9+21\,j_3+14\,j_3^2\right)\left(-\left(3^{\frac{1}{8}}\,(j_3\,(2+j_3))^{\frac{1}{8}}\right)+\left(-3+4\,j_3+4\,j_3^2\right)^{\frac{1}{8}}\right)^2 \times\left(-2\,(j_3\,(2+j_3))^{\frac{1}{8}}+\sqrt{2}\,3^{\frac{1}{8}}\left(-3+4\,j_3+4\,j_3^2\right)^{\frac{1}{8}}\right)^2\left((j_3\,(3+2\,j_3))^{\frac{1}{8}}-\left(-6+9\,j_3+6\,j_3^2\right)^{\frac{1}{8}}\right)^2$$

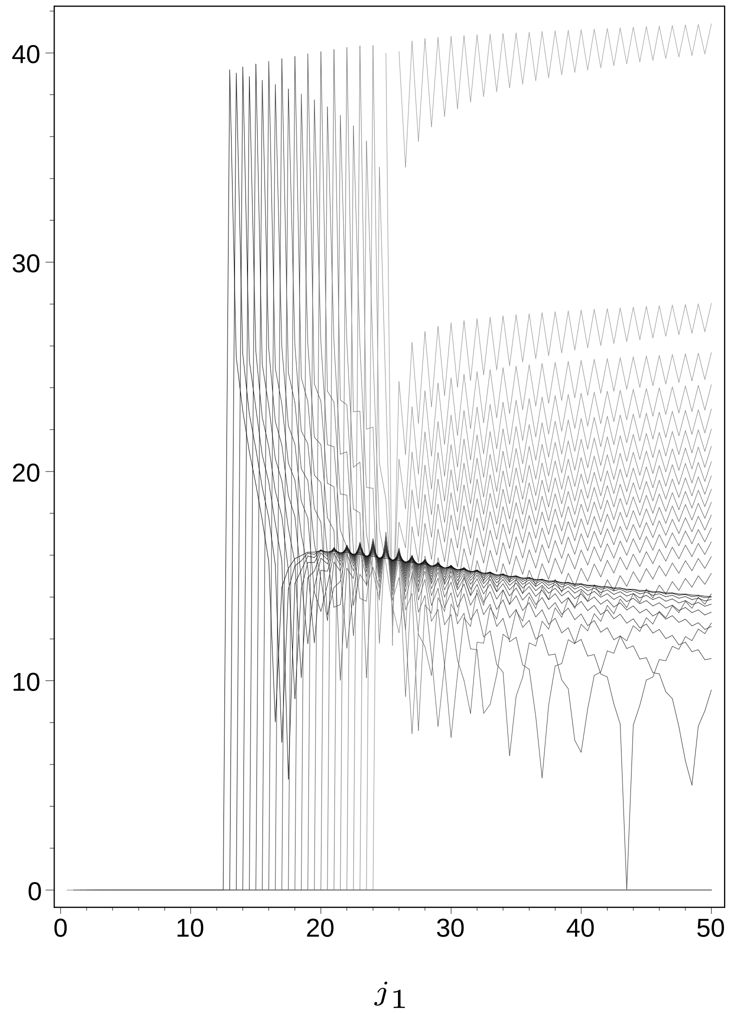


Asymtotically this increases as

$$\| \hat{H}^{(grav)}_{kin,v} \| \propto 1.06 \cdot 10^{-6}\; {j_3}^{\frac{3}{2}}$$

Plot of the Result: $30 + \ln\left[\frac{\left\| \hat{H}^{(grav)}_{kin,v} \right\|}{const}\right] \qquad j_3 = \frac{51}{2}$





Boundedness of Inverse Scale Factor Expectation Values I

- Complexifier Coherent States

$$\psi_{(A_0,E_0)} = \sum_\gamma \psi_{\gamma,(A_0,E_0)} := [e^{-\hat{\mathbf{C}}/\hbar}\,\delta_{A'}]_{A'\mapsto A^{\mathbb{C}}(A_0,E_0)}$$

where $A^{\mathbb{C}} := \sum_{n=0}^{\infty} \frac{i^n}{n!}\{\mathbf{C},A\}_{(n)}$ and $\{\mathbf{C},A\}_{(0)} = A$, $\{\mathbf{C},A\}_{(n+1)} = \{\mathbf{C},\{\mathbf{C},A\}_{(n)}\}$,
moreover $\delta_{A'}(A) = \sum_c T_c(A')\overline{T}_c(A)$ with $c = \gamma, \vec{j}, \vec{I}$
(Compare with harmonic oscillator: $\mathbf{C}_{osci} = \frac{p^2}{2m\omega}$, classicality parameter $s = \frac{\hbar}{m\omega}$),
should be adapted to Hamiltonian C, but that's too complicated

---

$\rightsquigarrow$ Area Operator ($\sim E^2(S)$) as complexifier

$$\psi_{\gamma,(A_0,E_0)} = \prod_{e\in E(\gamma)} \psi_{e,(A_0,E_0)}$$

where $\psi_{e,(A_0,E_0)}(A) = \sum_{2j=1}^{\infty} (2j+1)\, e^{-\frac{t(e)}{2}j(j+1)}\, \mathrm{Tr}(\pi_j(g_e(A_0,E_0)\, h_e(A)^{-1}))$,

$$g_e(A_0,E_0) \approx e^{\frac{i}{2}\tau_j E_0^j(S_e)} h_e(A_0)$$

Boundedness of Inverse Scale Factor Expectation Values II: Singularity Avoidance

- $U(1)^3$-CS Calculation (results qualitatively equal to full $SU(2)$-calc, T. Thiemann, O. Winkler GCS I-III)

  - Study $\quad {}^{(r)}\hat{e}^j_e(v) = \mathrm{Tr}\Big[\ \tau_j\ \hat{h}_e\ \big[\ \hat{h}_e^{-1}\ ,\ \hat{V}_v^r\ \big]\ \Big] \qquad$ for $\ SU(2)$

    $\hat{q}^j_e(v,r) = \mathbb{i}\ \hat{h}^j_e\ \big[\ (\hat{h}^j_e)^{-1}\ ,\ \hat{V}_v^r\ \big] \qquad$ for $\ U(1)^3$

    $$\text{where } \hat{V}_v^r = \ell_P^3 \Big[\Big|Z \sum_{I<J<K} \epsilon(I,J,K)\ \hat{q}_{IJK}\Big|\Big]^{\frac{r}{2}}$$

    $$\hat{q}_{IJK} = \epsilon_{ijk}\ X^i_I X^j_J X^k_K \qquad SU(2)$$

    $$\hat{q}_{IJK} = \epsilon_{ijk}\ n^i_I n^j_J n^k_K \qquad U(1)^3$$

  - Coherent States

    $$\Psi^{(v)}_{m,\gamma} = \prod_{\substack{e\in E(v)\\ j=1,2,3}} \sum_{n^j_e\in\mathbb{Z}} \mathbb{e}^{-\frac{t(e)}{2}[n^j_e]^2} \Big[h^j_e\big(Z(m)\big)h^j_e(A)^{-1}\Big]^{n^j_e}$$

Boundedness of Inverse Scale Factor Expectation Values II: Singularity Avoidance

- Upper Bound for Expectation Value at M-Vertex in $U(1)^3$-CS

$$\left\langle\ \Psi^{(v)}_{m,\gamma}(A)\ \middle|\ \hat{H}^{(grav)}_{kin,v}\ \middle|\ \Psi^{(v)}_{m,\gamma}(A)\ \right\rangle \leq$$
$$\leq \frac{(36)^2}{(\ell_P)^3}\frac{(9M)^6|Z|^{\frac{3}{2}}\left[\frac{2\mathbb{A}}{T^2}\right]^6}{\prod\limits_{I,i}[1+K^i_{t(I)}]}\left[\left[\frac{3Mp}{4T}\right]^6+\sum_{n=1}^{6}\frac{6!}{(6-n)!\ n!}\left[\frac{3Mp}{4T}\right]^{6-n}\prod_{l=1}^{n}\left[\frac{3M+2(l-1)}{4}\right]\right]$$

where $T = \min\limits_I T_I$, $T_I = \sqrt{t(e_I)}$, $t(e_I) := \frac{\hbar\kappa}{4L^3} f_{e_I}$

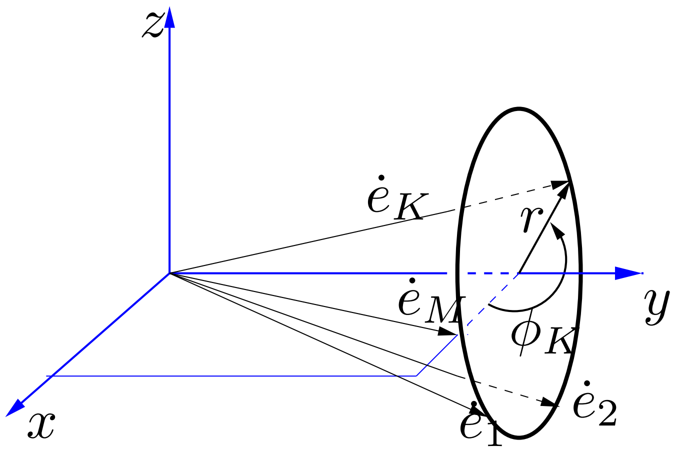
$$p = \max_{I,i} p^i_I,$$

$$p^i_I \approx \frac{1}{L^3}\int_{e_I} E^b_i\big(e_I(t)\big)\ q^0_{ab}\big(e_I(t)\big)\ \dot{e}^a_I(t)\ dt \qquad \mathbb{A} = 1 + \frac{p}{T}$$

- Expectation Value is finite, no matter if peaked at classically singular $(E = 0)$ configuration)

Boundedness of Inverse Scale Factor Expectation Values III: Singularity Avoidance

- Explicit edge Dependance - Problem with 'pathological edge configs:



- Ruled out by gauge invariance:

$$\hat{V} = \sqrt{\left| Z \cdot \sum_{I<J<K<M} \left[\epsilon(IJK) - \epsilon(JKN) + \epsilon(IKN) - \epsilon(IJN) \right] \; \hat{q}_{IJK} \right|}$$

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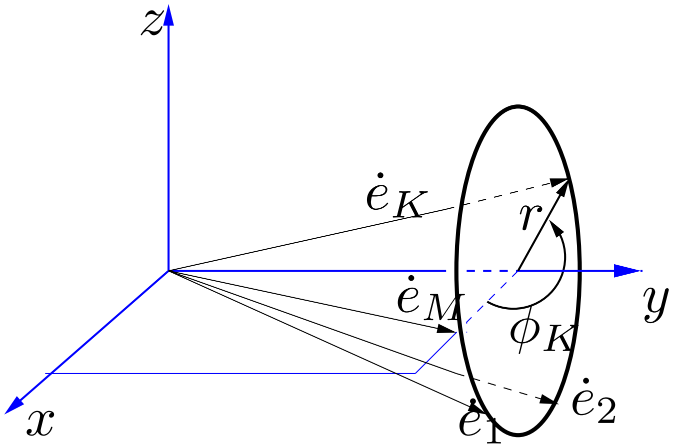

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! Remaining Combinatorical Issue with Sign Factors and Volume Operator at M-valent vertex v
$\rightarrow$ work in progress (with D. Rideout)

Summary

- Analogon of Inverse Scale Factor not a bounded operator in full LQG

  but

- $U(1)^3$-CS calculation reveals boundedness of the expectation value

  encouraging, but not sufficient to prove quantum mechanical absence of singularity $\rightsquigarrow$

  Need to construct (approximately) physical states which do not suffer from unbounded behaviour of hamiltonian. Construction in analogue to gr-qc/0507029 P. Singh, K. Vandersloot...

Current Work $\rightsquigarrow$ Outlook

- Singularity Avoidance

	Ideal Approach	Approximate Approach
(i)	$\mathcal{H}_{phys}$	kin. CS with $\langle \Psi, \hat{C}\Psi \rangle \approx 0$
(ii)	Phys. $\hat{H}$ via Partial Obs.	$\hat{H}_{approx}$ via finite power series expansion of Partial Obs.
(iii)	CS preserved by $\hat{H}$	CS preserved by $\hat{H}_{approx}$, if too difficult, take CS peaked on classically singular initial data
(iv)	Evolve expect. values	Evolve expect. values

- Generalize Result: $\longrightarrow$ gauge invariant 4-vertex

2 Cosmology: Simplest Example FRW + Scalar Field

- Demand: Spatial homogeneity, isotropy →fixes form of metric and line element

$$ds^2 = -d\tau^2 + a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\Theta^2 + \sin^2\Theta d\phi^2\right)\right]$$

- Scalar field ϕ: (Einstein's)equations for the scale factor (dep. on potential of $V(\phi)$)

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N\left(\frac{\pi^2}{2a^6} + \frac{a^3V(\phi)}{a^3}\right) - 3\frac{k}{a^2} \qquad \dot{\pi} = -a^3\frac{dV(\phi)}{d\phi} \qquad \text{with } \pi = a^3\dot{\phi}$$

- Look at solutions close to $a = 0$

assume: $\lim_{a\to 0} a^3V = 0$ and $\lim_{a\to 0} a^3\frac{dV(\phi)}{d\phi} = 0$, $V(\phi)$ polynomial in ϕ

$\rightsquigarrow$ $\dot{\pi} \ll 1$ and $\pi \sim \mathit{const}$ near $a = 0$.

$\rightsquigarrow$ near $a = 0$: need to consider only the highest inverse power of a in Friedmann Eqn.(neglect all other terms)

$\rightsquigarrow$ $\boxed{\phi \propto \ln[\tau]}$ and $\boxed{a^3\phi^n \propto \tau\ln[\tau]^n}$, thus the assumption was justified

Quantization Ambiguities ?

- Classical Identity: $\det E = \operatorname{sgn}\Big[\det(e)\Big]\det(q)$

$$\text{with } \det(q) = \Big[\det(e)\Big]^2 > 0$$

- Therefore:

$$V = \int_R d^3x\sqrt{\det(q)} = \int_R d^3x\sqrt{|\det(E)|}$$

$$= \int_R d^3x\ |\det(e)| = \int_R d^3x\ \operatorname{sgn}[\det(e)]\,|\det(e)|$$

Consistency of Flux-Quantizaion (see Kristina's talk, paper [gr-qc/0507036]) $\rightsquigarrow$

$$\operatorname{sgn}[\det(e)] \stackrel{!}{=} \operatorname{sgn}[\det(E)] = \operatorname{sgn}[\hat{Q}] \qquad \text{where } \hat{V}_{AL} = \sqrt{|\hat{Q}|}$$