Causal Set Phenomenology

Fay Dowker

Imperial College, London

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Plan

1. Introducing the Causal Set (Definition and Hypothesis)

'tHooft; Myrheim; Bombelli, Lee, Meyer and Sorkin

2. Effects of discreteness on the continuum (We need evidence)

FD, Henson, Sorkin

A. A model of detector response to scalar field source

B. A model of massive particles

3. Could high energy cosmic rays be evidence?

4. Conclusions

Causal Sets

A causal set (or causet for short) is a set of elements, C , with a binary relation, $\prec$, called "precedes" which satisfies the following axioms:

1. Transitivity: if $x \prec y$ and $y \prec z$ then $x \prec z$, $\forall x, y, z \in C$;

2. Non-circularity: if $x \prec y$ and $y \prec x$ then $x = y$ $\forall x, y \in C$ (non-circularity);

3. Local finiteness: for any pair of fixed elements x and z of C, the set $\{y | x \prec y \prec z\}$ of elements lying between x and z is finite.

Of these axioms, the first two say that C is a partially ordered set or poset and the third makes the set discrete.

The Causal Set Hypothesis

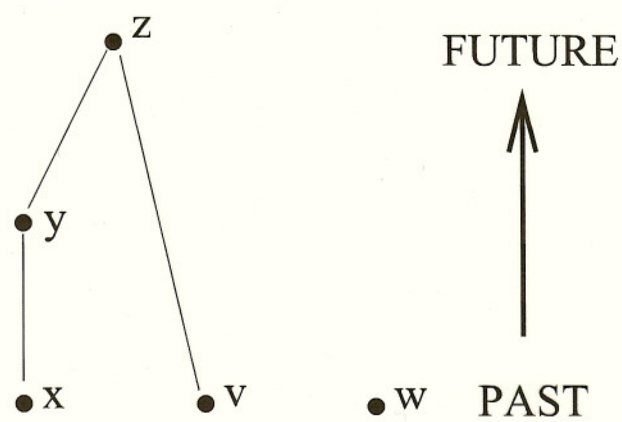
The deep structure of spacetime is a causal set

The discrete elements of the causal set are related to each other only by the partial ordering that corresponds to a microscopic notion of before and after, and the continuum notions of length, space and time arise only as approximations at large scales. Just as ordinary matter appears smooth and continuous on large scales but is really made of atoms, so it is proposed spacetime appears continuous to us but is fundamentally discrete.

The number of causal set elements in a region gives rise in the continuum approximation to what we experience as the spacetime volume of the region (in fundamental units close to Planck units) and the order gives rise in the continuum approximation to the spacetime causal order.

Powerful theorems (by Hawking, Malament and Levichev) in continuum causal analysis tell us that the volume and causal structure are enough to recover the full geometry and this means that a causal set can contain enough information to be able to be well-approximated by a continuum spacetime.

A nice way to represent a causal set is by drawing its Hasse diagram. For example:



Elements are vertices and relations are edges. Element x precedes ("is in the past of") y and y precedes z. Transitivity implies that x precedes z so that relation does not need to be drawn in. The irreducible relations are called links. Element w is unrelated to any other.

In a causal set that could be our visible universe there would be of the order of 10^{240} elements with a correspondingly large and complex web of relations.

Phenomenological effects of discreteness

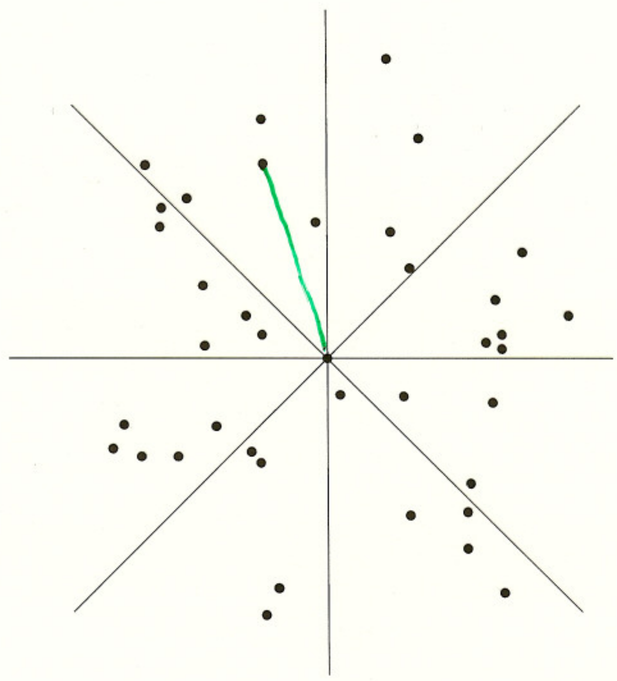
Assuming that causal set theory is right, the spacetime we observe around us is only an approximation to deeper level of reality that is a causal set. One way to investigate potential effects of that underlying discreteness is to make models of matter, fields, particles etc. on the background of a causal set that could have our observed spacetime (Minkowski space for definiteness) as an approximation.

Question: What could that causet be?

Rough Answer: A causet that arises by discretising Minkowski spacetime "faithfully".

Detailed Answer: A causet that is produced (with high probability) by a Poisson process of "sprinking" elements at random into Minkowski space so that the mean number of elements falling in any region is given by the volume of the region (in near-Planck units). The sprinkled elements are endowed with the order relations induced by the Minkowski causal order.

A Poisson distribution of points in 1+1 Minkowski



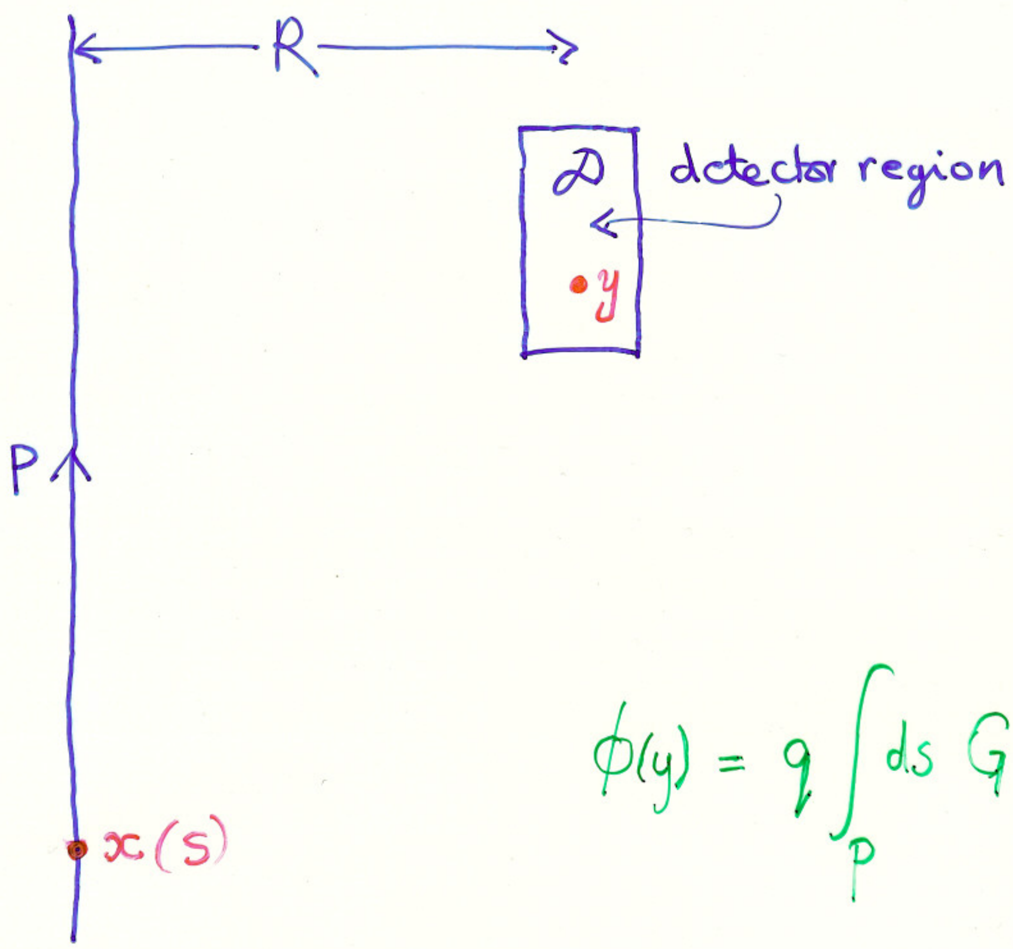
IMPORTANT POINT: This distribution is Lorentz invariant – it is uniform in any frame. Causal sets are Lorentz invariant.

[Claim: fundamental spacetime discreteness + Lorentz invariance leads more or less uniquely to causal sets Henson]

A. Model of source-detector response

Consider the following setup

SPACETIME – 4D MINKOWSKI



$$\phi(y) = q \int_P ds \, G(x(s), y)$$

world line of source, charge q, of massless scalar field

CONTINUUM

In the continuum, we model the output F of the detector as the integral of the field's value over the region $\mathcal{D}$, i.e.

$$\begin{aligned} F &= \textstyle\int_{\mathcal{D}} d^4y\, \phi(y) \\ \phi(y) &= q \textstyle\int_P ds\, G(x(s), y) \end{aligned}$$

where P is the worldline of the source, q is the charge of the source and s is proper time along P.

$G(x, y)$ is the retarded Green's function and is equal to

$$\begin{aligned} G(x,y) = & \ \tfrac{1}{2\pi}\delta(|x-y|^2) \text{ if } y \in J^+(x) \\ & \ 0 \qquad \text{otherwise} \\ = & \ \tfrac{1}{4\pi r}\delta(y^0 - x^0 - r), \end{aligned}$$

where r is the spatial distance from x to y.

A short calculation shows that the field $\phi(y)$ is roughly constant over the detector region and is equal to $q/4\pi R$ as expected.

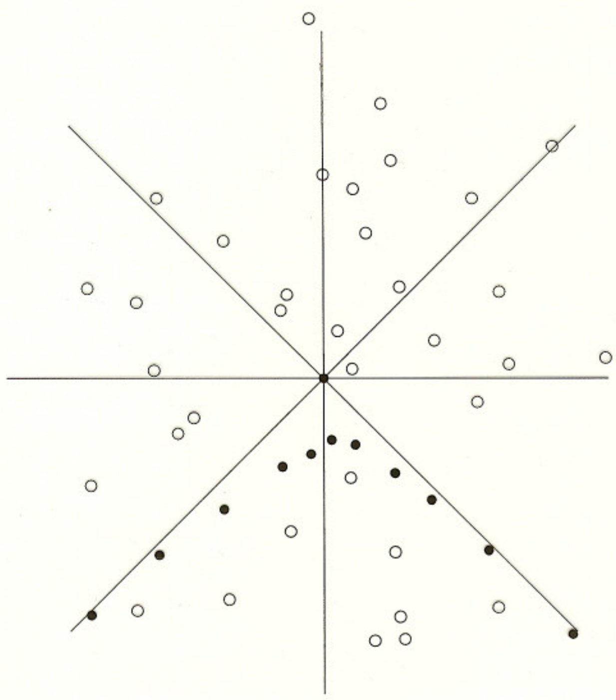
In a causet model of this same situation a sprinkling C_S of 4D Minkowski space replaces the continuum background. The source is now modelled as the subset, $\tilde{P}$, of the causet which lies in a tube of unit Planck cross section – in the rest frame of the detector – centred on the continuum worldline of the source.

The detector region D is replaced by the set $\tilde{D}$ of all elements that were sprinkled into that region of spacetime. A scalar field is a function from the elements of the causet to the Reals.

We need a causet analogue of the retarded Green's function $G(e_i, e_j)$ which is zero unless e_j is "on the past light cone of e_i". There is such a thing [Daughton, Pullin, Salgado, Sorkin]:

& Woolgar

$G(e_i, e_j) = \sqrt{6}/12\pi$ if $e_i \prec e_j$ and the relation between them is a link i.e. there's no other element between them. And it's zero otherwise. This works because the past links from a given element are distributed like this:



The output of the detector in this discretisation is $\tilde{F}$ a sum over all elements sprinkled into the detector region:

$$\begin{aligned} \tilde{F} = & \quad \textstyle\sum_{e_j \in \tilde{D}} \phi(e_j) \\ \phi(e_j) = & \quad q \textstyle\sum_{e_i \in \tilde{P}} G(e_i, e_j) \end{aligned}$$

where q is the charge.

This is the total number of links between elements in $\tilde{P}$ and in $\tilde{D}$ and it is something we know how to estimate because we know the statistics of the sprinkling.

The expected value of the field at e_j is

$\langle\phi(e_j)\rangle = q\sqrt{6}/12\pi$ times the expected number of links between e_j and elements sprinkled into region P.

To calculate this, imagine P broken up into tiny volumes, d^4x. The probability that there'll be an element in d^4x at x is proportional to d^4x and the probability that it will be linked to e_j at point y is the probability that the interval between x and y is empty of sprinkled points. That probability is $e^{-I(x,y)}$ where $I(x,y)$ is the spacetime volume of the interval.

Thus we need to calculate

$$\int_P d^4 x e^{-I(x,y)}$$

and it gives

$$\langle \phi(e_j) \rangle = \frac{q}{4\pi R} + O(1/R^3)$$

so the deviation from the continuum result is truly tiny. But what about fluctations about the mean?

The statistical nature of the correspondence between the continuum and the causal set will mean that there are always deviations from mean values (a crucial ingredient in the derivation of the measured value of the cosmological constant from causal set theory) The question is, how large will they be? Usually they are very small (e.g. 10^{-120} for the cosmological constant). And so it turns out in this case.

In general, if the mean of some quantity is a large number N (here N is the number of links between two subsets of a causet) and there are no correlations, then the standard deviation is $\sqrt{N}$ and the "signal to noise ratio" goes like $\sqrt{N}$.

For our link counting, there are some mild correlations to be taken into account, but the bound on the s.d. is still roughly $\sqrt{N}$. For a detector of nuclear size with a time resolution of $10^{-15}s$ and $R = 1MParsec$ it means a signal to noise ratio of 10^{15}.

That's good agreement: TOO GOOD!!

(Maybe the discrete D'Alembertian can do worse!)

B. Swerves

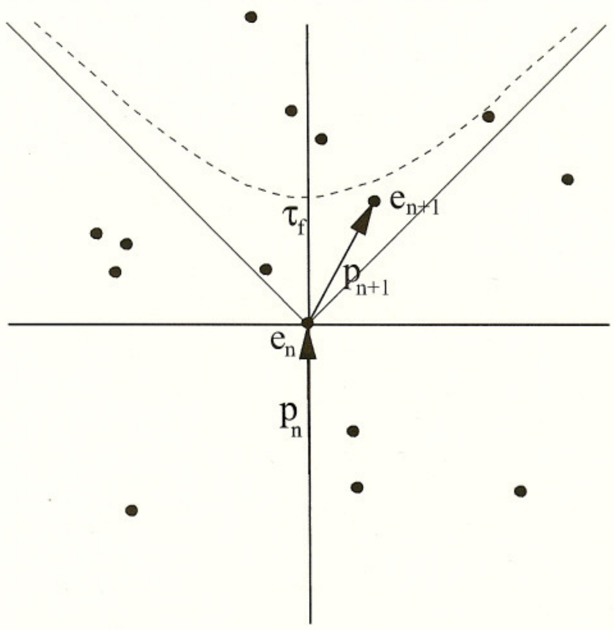
The following is a model (due to Henson) of a massive particle moving on a causal set. The idea is that a particle cannot follow an exact straight line geodesic because of the underlying discreteness but it does the best it can do and still be approximately Markovian.

We call the effect swerves, after Lucretius:

"The atoms must a little swerve at times – but only the least, lest we should seem to feign motions oblique, and fact refute us there."

Swerves occur when a particle tries to move along a straight line geodesic but cannot because the underlying reality is discrete.

Consider a sprinkling into Minkowski spacetime (1+1 dimensions are shown, really it is 3+1)



At each stage: particle moves from the element, e_n, where it is to one in its causal future, minimising the change in its momentum, within a proper time τ_f. Repeat.

Choosing τ_f to be large compared with the Planck scale means that the swerving effect is small: the change in momentum is small at each step. But over many steps it can add up.

Due to the randomness of the sprinkling, the momentum is going on a Lorentz invariant random walk on the mass shell. In the hydrodynamics limit of a large number of steps this is described by a "Brownian motion", governed by a diffusion equation on the mass shell:

$$\frac{\partial \rho}{\partial \tau} = k\nabla^2 \rho - \frac{1}{m} p^\mu \frac{\partial}{\partial x^\mu} \rho$$

laplacian on mass shell H^3

$$\rho \equiv \rho(x^\mu, p^\nu ; \tau)$$

This is the unique (up to a free parameter) Poincaré invariant, relativistically causal diffusion on $M^4 \times H^3$. The free parameter is the diffusion constant. It is an Ornstein-Uhlenbeck type stochastic process: a diffusion in momentum that drives a secondary process in spacetime.

Although the discrete model was rather ad hoc (and also depends on the continuum, so it can't be fundamental) the uniqueness of this diffusion process means that any discrete model that is Lorentz invariant and causal will end up giving the same continuum model.

Dudley 1965; FD, Henson, Sorkin

No conservation of energy:

Swerves produce a statistical acceleration of particles: an initial distribution that is peaked at zero energy in some frame will spread so that later the distribution has support at high energies in that frame. Is there any evidence that particles do get spontaneously accelerated?

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Cosmic Rays

We have been collecting data on high energy cosmic rays for many decades and this is what it looks like:

[Page 6 of Anchordoqui et al
http://arxiv.org/abs/hep-ph/0206072]

from: Anchordoqui et al 2002

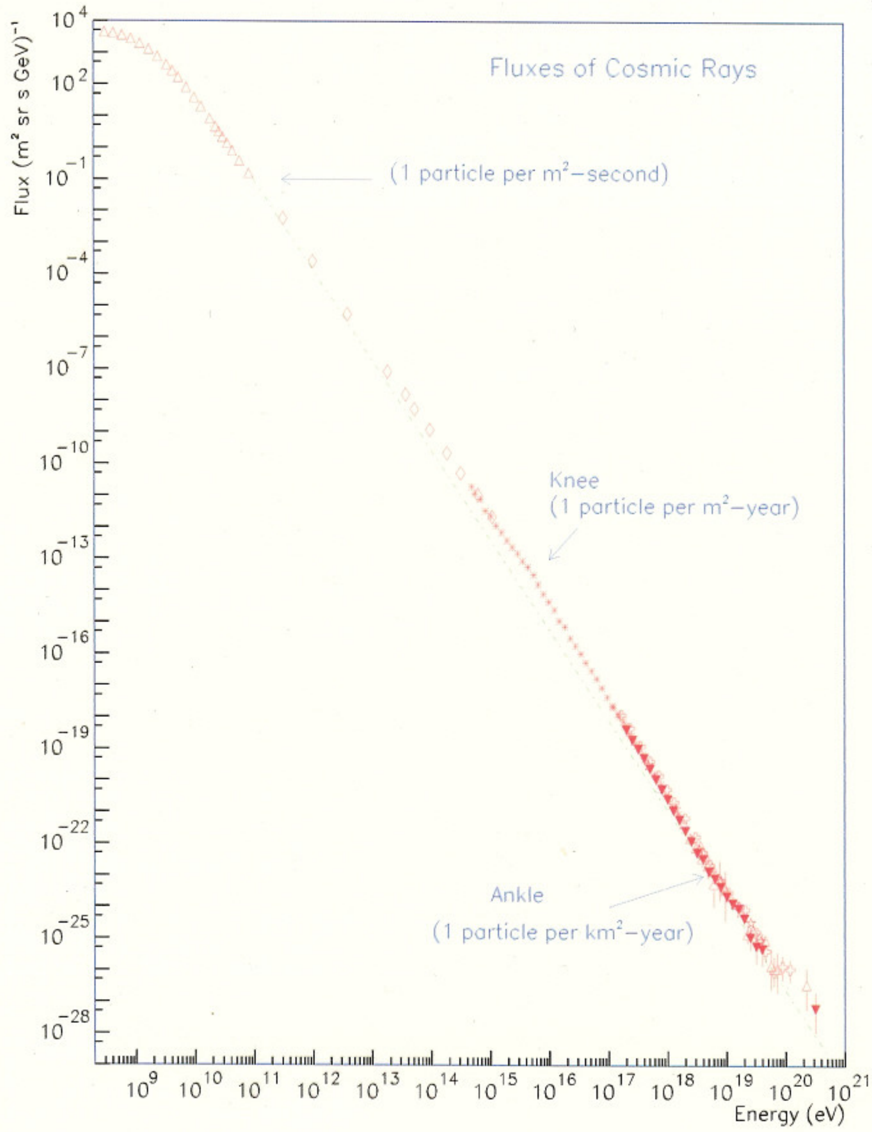

FIG. 1: Compilation of measurements of the differential energy spectrum of CRs. The dotted line shows an E^{-3} power-law for comparison. Approximate integral fluxes (per steradian) are also shown [18].

Their origin is a still a mystery. In the review paper cited before the following were listed as just some of the proposed sources:

Supernovae explosions, Large scale Galactive wind termination shocks, Pulsars, Active Galactic Nuclei, BL Lacertae, Spinning supermassive black holes, Large scale motions and related shock waves resulting from structure formation, Relativistic jets produced by powerful radiogalaxies, Electric polarisation fields in plasmoids produced in planetoid impacts on neutron star magnetospheres, Magnetars, Starburst galaxies, Magnetohydrodynamic wind of newly formed strongly magnetized neutron stars, Gamma ray burst fireballs, Strangelets, Hostile aliens with a big cosmic ray gun.

Interestingly, the mechanism for the acceleration of the particles within this myriad of sources is the same: Fermi acceleration which is a statistical acceleration of charged particles scattering off random magnetic field irregularities: like swerves except that swerves occur in the vacuum.

So could swerves be responsible for the acceleration required to produce a cosmological background of high energy protons, say, that could account for the cosmic ray data?

Unfortunately, no. The Lorentz invariance of the process means that a proton that already has energy 10^{18}eV, say, in the cosmic frame, sees the lifetime of the universe as only about ten years in its rest frame. If the diffusion constant is such that this proton has a reasonable chance of doubling its energy in the lifetime of the universe then it would be so big that boxes of hydrogen gas in the lab would spontaneously heat up at a measureable rate.

Speculations

So this simple model of swerves can't explain the cosmic ray data. But the data just cries out for a universal, cosmological acceleration mechanism and it seems worth trying to improve on the model.

One improvement would be to make it quantum mechanical. We don't yet have the same overarching framework for quantum random walks that we do in the classical case. But results from quantum information theory suggest that in contrast to classical diffusion in which the variance of a Gaussian distribution grows as t, in a quantal diffusion the variance would increase as t^2 [Carlos Perez]. If we can argue that hydrogen in the lab is classical whereas protons in space are quantal, we may be able to finesse the laboratory constraints on the diffusion constant and produce a model that can provide a cosmological acceleration mechanism for cosmic rays.

Conclusion

The concrete kinematics of causal set theory allows us to do phenomenology. Lorentz invariance is a key feature in all models.

A simple model of source-detector coupling shows that a scalar field on a causal set background can reproduce continuum results very well – TOO WELL!

The model of swerves has new and potentially measureable effects on particle propagation.

Perhaps quantum swerves can produce a cosmological background of high energy cosmic ray primaries to match the observed flux. High energy cosmic rays would then be the Brownian motion of our age: the phenomenon that in the future will convince us of the fundamental discreteness of spacetime itself.