More is Classical

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Outline

- Talk in the foundations of quantum mechanics
- Aim is to show that quantum mechanics is *deterministic*
- Show what this means for the search of a quantum theory of gravity

The measurement problem

System: $|a\rangle \ |b\rangle$

Apparatus: $|A\rangle \ |B\rangle \ |N\rangle$

Measurement:

$$|a\rangle|N\rangle \longrightarrow |a\rangle|A\rangle \qquad |b\rangle|N\rangle \longrightarrow |b\rangle|B\rangle$$

Linearity:

$$(\alpha|a\rangle + \beta|b\rangle)|N\rangle \longrightarrow \alpha|a\rangle|A\rangle + \beta|b\rangle|B\rangle$$

not observed

The solution: Overview

The solution requires two steps:

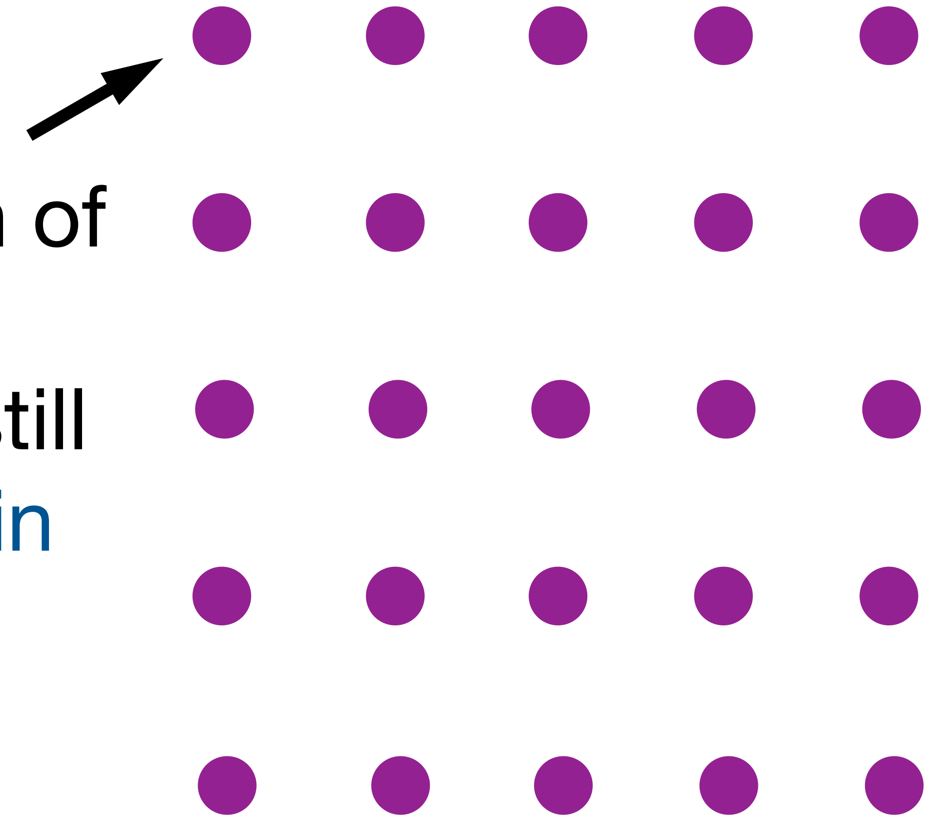
- *Symmetry broken* states are *classical* states (Hepp, Anderson, Laughlin, ...)
- Symmetry breaking introduces *uncertainty*, hence the *probabilistic* nature of quantum mechanics.

Symmetry broken states are classical states

Symmetry broken states have two properties that make them appear classical:

- Order
- Rigidity

Order & Rigidity



Position of single atoms still uncertain

Average Position of the atoms known to high accuracy

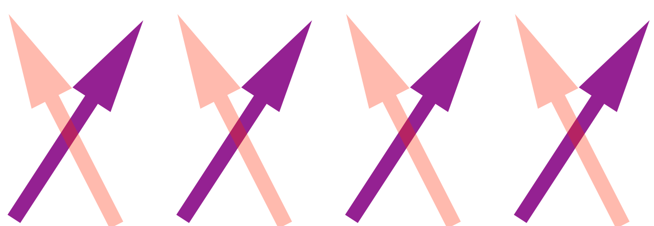
Rigidity: The whole systems reacts when one moves part of it

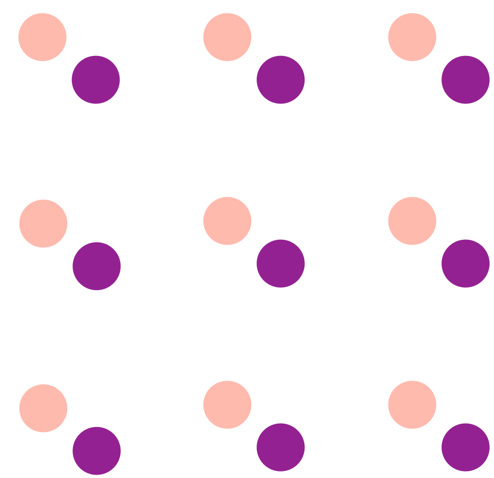
The symmetry breaking problem

In typical Hamiltonians,

$$H = \sum_{\langle i,j \rangle} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \quad H = \sum_i \frac{\boldsymbol{p}_i^2}{2m_i} + \frac{1}{2} \sum_{i \neq j} V(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

nothing singles out a particular ground states.





How then is it chosen? Why do we not have a '*symmetry breaking problem*' ?

Solving the symmetry breaking problem

- The symmetry broken state has *lower energy* and *entropy*.
- Symmetry breaking can only occur when the system is coupled to an *environment*.
- The environment introduces a *stochastic* element.
- System and apparatus are not ergodic.

$$|N\rangle$$

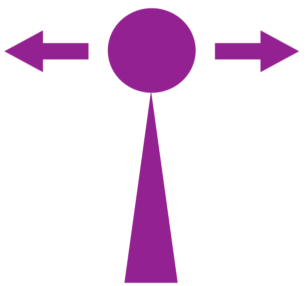
is only symmetric in an *ergodic* sense.

The roles of the environment

- Put the system close to a symmetry breaking point.
- Serve as a dump for energy and entropy.
- Provide the *random* push into one of the possible states.

The measurement problem

- Measurement should *amplify* the signal from the system.
- Such an amplifier is a system about to undergo symmetry breaking.



- Catch: Amplification introduces *uncertainty* inherent in the symmetry breaking process.
- Measurement process becomes *stochastic*.

What went wrong?

Why does

$$|a\rangle|N\rangle \longrightarrow |a\rangle|A\rangle \quad |b\rangle|N\rangle \longrightarrow |b\rangle|B\rangle$$

not imply

$$(\alpha|a\rangle + \beta|b\rangle)|N\rangle \longrightarrow \alpha|a\rangle|A\rangle + \beta|b\rangle|B\rangle$$

?

We have not taken into account the *environment*. The new experiment is a new role of the dice. *Linearity does not apply*.

The Born rule

- If this is the story we should be able to derive the *Born rule*.
- Rub: Symmetry breaking is hard. Very little is known analytically.
- Solution: Use arguments by S. Saunders (also Deutsch & Wallace). Here they translate into assumptions about the Hamiltonian.

The derivation

Want to calculate the expectation value

$$V(\psi, \Omega) = p_1 \Omega(1) + p_2 \Omega(2) + \text{etc.}$$

and show that the p_i are given by the Born rule:

$$p_i = |\langle i|\psi\rangle|^2$$

Idea: The symmetries of the Hamiltonian are also symmetries of $V(\psi, \Omega)$.

The derivation II

Example: Two state system

$$\boldsymbol{H} = \sum_i \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_i$$

Symmetries:

$$\begin{pmatrix} \alpha & 0 \\ 0 & \bar{\alpha} \end{pmatrix} \in \mathrm{SU}(2) \qquad F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$V(U_\alpha \psi, \Omega) = V(\psi, \Omega) \quad V(U_F \psi, \Omega) = V(\psi, -\Omega)$$

The derivation III

Special case: $\psi = \alpha|a\rangle + \beta|b\rangle$ with $|\alpha| = |\beta|$

$$V(\alpha|a\rangle + \beta|b\rangle, \Omega)$$

$$\overset{\text{(flip)}}{=} \quad V(\alpha|b\rangle + \beta|a\rangle, -\Omega)$$

$$\overset{\text{(phase)}}{=} \quad V(\beta|b\rangle + \alpha|a\rangle, -\Omega)$$

From this follows the Born rule for this case:

$$p_1 = p_2 = \frac{1}{2}$$

Lessons for quantum gravity

- No coherent states
- Dynamics or die
- Do not quantize
- Hard to find symmetry broken states
- Space is emergent

Lesson 1: No coherent states

- Classical states of quantum systems are *not* given by coherent states.
- The idea only worked (sort of) for one system: the harmonic oscillator.
- How would it work for a large collection of quantum systems? Is there a conservation of coherence? ...

Lesson 2: Dynamics or die

- The classical symmetry broken states are states singled out by the *dynamics* of the system.
- There is no knowledge of the symmetry broken states in the kinematics.

Lesson 3: Don’t quantize

- The quantization of the classical theory describing the degrees of freedom above the symmetry broken state does not give back the fundamental theory.
- Connection to Ted’s gravity from black hole thermodynamics?

Lesson 4: Hard to solve

- It is hard to find the symmetry broken states with analytical methods.
- There are no cook book procedures (renormalization group etc.) that give those states.

Lesson 5: Space is emergent

- Quantum mechanics looks so strange to us because we look at the quantum world with our classical eyes. These are used to properties like position that only *emerged* on the classical level but have no meaning on the quantum level.
- Space is not fundamental.
- Do not construct:

$$\int dg \exp(\int d^4 x R)$$

The fundamental theory has no x's and R's

Conclusion

- Physics of the 20th century can be viewed as the story of us coming to terms with symmetry breaking.
- Rigidity made Cartesian space look good. Special and general relativity were required to see beyond this.
- Symmetry breaking veiled quantum mechanics from us.
- Story is not finished ...