TO BE

OR NOT

OBSERVABLE

S.H. with Oliver Winkler

$\hat{\pi}$ $\hat{\pi}$, N.B.

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$\mathcal{P}_\Phi$ OWER SPECTRUM

$\mathcal{C}$ FRW background:

$$ds^2 = dt^2 - a^2(t)\, \delta_{ij}\, dx^i dx^j$$

Φ on $\mathcal{C}$ FRW background:

$$H_s = \int \mathrm{vol}\, \rho_s(\Phi, \pi, a)$$

Vacuum 'proposal':

$$A_s(\Phi, \pi; k, t_i)\, |\Omega, t_i\rangle = 0 \quad \forall k$$

Solution:

$$|\Psi[\Phi(k, t_i), t]|^2 \propto \exp\left\{ - \frac{|\Phi(k, t_i)|^2}{\mathcal{P}_\Phi(k, t)} \right\}$$

$\uparrow$

minimal uncertainty
wave function

CONNECTION TO dS-QFT

Orthodox calculation:

$$\Phi(k,t) = W(k,t)\, A(k,t_i) + h.c.$$

Choice of vacuum:

$$A(k,t_i)\,|\Omega,t_i\rangle = 0 \quad \forall k$$

$$\widehat{=}$$

minimal uncertainty of $\Delta\Phi\,\Delta\Pi$
lowest energy state
instantaneous Minkowski vacuum

$$\supset$$

Bunch-Davies vacuum : $t_i \longrightarrow -\infty$

Power spectrum:

$$\mathcal{P}_\Phi = \frac{k^3}{2\pi^2}\langle\Omega|(\Phi^\dagger\Phi)(k,t)|\Omega\rangle\Big|_{k=aH}$$
$$= \left(\frac{H}{2\pi}\right)^2$$

$\mathcal{P}_\Phi$ SUPERSTAR

For a cosmological fluctuation $F(k,t)$

(e.g. $\frac{\delta T}{T}$, $\frac{\delta \rho}{\rho}$, ...)

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in the linear regime :

$$\mathcal{P}_F(k,t) = \mathcal{P}_\Phi(k,t_*)\, T_F(k,t-t_*)\, D_F(k)$$

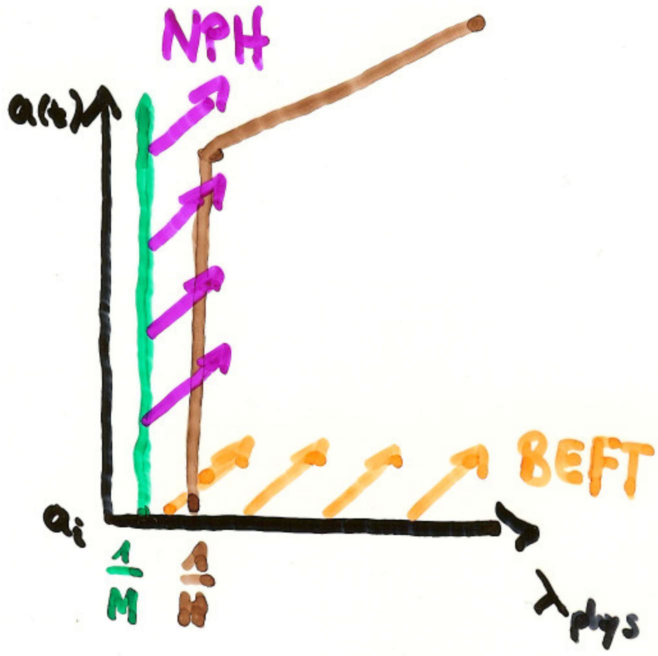
$\mathcal{P}_\Phi(k,t_*)$	$T_F(k,t-t_*)$	$D_F(k)$
initial spectrum	gravitational evolution	damping factor
model of inflation	cosmological perturbation theory	nonequilibrium physics
QFT of Φ	General Relativity	QFT of F

BOUNDARY PROPOSALS

Choice of a vacuum in the Hamilton formalism

$\Longleftrightarrow$

Choice of a boundary condition in the Lagrange formalism



$$\mathcal{P}_\Phi^{BEFT} = \mathcal{P}_\Phi^{BD}\left[1 + \left(\frac{H}{M}\right)^1 \frac{k}{a_i H}\sin\left(2\frac{k}{a_i H}\right)\right]$$

$$\mathcal{P}_\Phi^{NPH} = \mathcal{P}_\Phi^{BD}\left[1 + \left(\frac{H}{M}\right)^1 \frac{a_c M}{k}\sin\left(2\frac{a_c H}{k}\right)\right]$$

Φ IN ⟨QFRW⟩ COSMOLOGY

Hilbertspace : $\mathcal{H}_{QFRW} \otimes \mathcal{H}_{\Phi}$

Idea : $a^{-1} \xrightarrow[\text{field}]{\text{mean}} \langle QFRW | a^{-1} | QFRW \rangle$

$$H_S = \int \langle QFRW | \,\mathrm{vol}\, \rho_S \,| QFRW \rangle$$

$$\langle \rho_S \rangle = \frac{1}{2}\left(\langle a^{-3}\rangle \pi\right)^2 + \frac{1}{2}\left(\langle a^{-1}\rangle \nabla \Phi\right)^2 + V(\Phi)$$

Equations of motion :

$$\ddot{\Phi} - \left[3\frac{\langle \dot{a}\rangle}{\langle a\rangle} + 2\frac{\langle \dot{a^{-3}}\rangle}{\langle a^{-3}\rangle}\right]\dot{\Phi}$$

$$- \left(\langle a^{3}\rangle\langle a^{-3}\rangle\right)^2\left[\left(\langle a^{-1}\rangle \nabla\right)^2 \Phi - \frac{dV}{d\Phi}\right] = 0$$

$$\frac{\langle \dot{a}\rangle}{\langle a\rangle} = \frac{8\pi}{3} G_N \langle \rho_S \rangle$$

Limit : QFRW $\longrightarrow$ FRW ✓

<QFRW> CLOSE TO FRW

Large parameter:

$$x \overset{\text{Def}}{=} \frac{a}{a_i} \gg 1$$

Equation of motion:

$$\left(\frac{d^2}{dx^2} + \Omega^2(x)\right) \varphi(k,x) = 0$$

Normal frequencies:

$$\Omega^2 = \overbrace{\left(\frac{1}{x}\right)^2 \Bigg[\underbrace{\left(\frac{k/a_i}{H}\right)^2}_{\text{redshift}} \left(\frac{1}{x}\right)^2 - \underbrace{2}_{\text{Hubble friction}} \Bigg]}^{\text{classical}} +$$

$$\underbrace{+ \left(\frac{1}{x}\right)^2 \Bigg[\left(\frac{k/a_i}{H}\right)^2 \left(\frac{1}{x}\right)^2 \underbrace{f_k(x)}_{\text{redshift fluctuation}} - \frac{3}{2}\left(3 + x\frac{d}{dx} + \frac{3}{2} f_H(x)\right) f_H(x) \Bigg]}_{\langle \text{quantum} \rangle}$$

Hubble friction fluctuation

ASYMPTOTICS

Superhorizon: $\dfrac{k/a}{H} \sim \dfrac{R_H}{\lambda_{phys}} \ll 1$

$$\Phi_{sup}(x) = C_1(k, x_c)\, x^{-1} \exp\left(\frac{3}{2} f_H(x)\right) M\Big(-1, \frac{3}{2}, 3\underset{\underset{x^{-2}}{\approx}}{f_H(x)}\Big)$$

Subhorizon: $\dfrac{k/a}{H} \sim \dfrac{R_H}{\lambda_{phys}} \gg 1$

$$\Phi_{sub}(x) = \frac{1}{(2k)^{1/2}} \frac{1}{a} \frac{1}{(1+f_H(x))^{1/4}} \exp\left(\frac{3}{2} f_H(x)\right) \exp\left\{ i \frac{k/a}{H} \frac{(1+f_H(x))^{1/2}}{f_H(x)} \right\}$$

classical

Glueing:

$$C_1(k, x_c) \longrightarrow \frac{1}{(2k)^{1/2}} a_c^2$$

$\mathcal{P}_{\Phi}$ IN ⟨QFRW⟩ COSMOLOGY

$$\mathcal{P}_{\Phi}^{\langle QFRW\rangle}(k) = \left(\frac{H}{2\pi}\right)^2 \left(1 + \sqrt{8\pi}\,\frac{H}{M_{Pl}}\,\frac{1}{k}\right)$$

classical

order parameter

scale dependence

$$\left.\frac{\delta\mathcal{P}_{\Phi}^{\langle QFRW\rangle}}{\mathcal{P}_{\phi}^{FRW}}\right|_{max} = \sqrt{8\pi}\,\frac{H}{M_{Pl}}\left(\frac{M_{Pl}}{H}\right)^{1/3}$$

$$\approx 10^{-2}$$

observable

SUMMARY

- $\exists$ framework to predict quantum cosmological imprints on cosmological high precision observables

- $$\mathcal{O}\left[\frac{\delta P^{\langle QFRW\rangle}}{P^{FRW}}\right] = \frac{H}{M_{Pl}}$$

- $$\max \frac{\delta P^{\langle QFRW\rangle}}{P^{FRW}} \gtrsim 1\%_0$$

observable