QUANTUM BLACK HOLES: DYNAMICS.

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LOOPS 05

THE PROBLEM:

FIND FULL QT OF THE GRAVITY–SCALAR FIELD THEORY IN SPHERICAL SYMMETRY.
(ASYPT. FLAT SECTOR)

NON-TRIVIAL 1+1 DIM. FIELD THEORY

WHY?

* WHAT DOES QG COLLAPSE LOOK LIKE?

  CLASSICAL COLLAPSE WELL STUDIED NUMERICALLY (GOLDWIRTH-PIRAN, CHOPTUIK ...)

* WHAT IS A "QUANTUM BLACK HOLE"?

  SEEK THE RESULT OF A LONG TIME QUANTUM COLLAPSE EVOLUTION.

* DOES HAWKING RADIATION SHOW UP IN A SUITABLE APPROXIMATION?

  RESOLUTION OF TRANSPLANCKIAN MODE PROBLEM?

THIS PROBLEM WAS POSED BY UNRUH (1976) TO UNDERSTAND HAWKING RADIATION.

→ FULL GAUGE FIXING TO GET A REDUCED THEORY FOR THE SCALAR FIELD:

$$H[Q, P_Q] = \int_0^\infty dr\, f(Q, P_Q)\, e^{\int_\infty^r dr' g(Q, P_Q)}$$

"I PRESENT IT HERE IN THE HOPE THAT SOMEONE ELSE MAY BE ABLE TO DO SOMETHING WITH IT"

* USELESS HAMILTONIAN IS THE RESULT OF A RADIAL COORDINATE GAUGE FIXING.

OUR STRATEGY:

* FIX A TIME GAUGE TO BE ABLE TO ADDRESS AT LEAST SOME DYNAMICAL QUESTIONS UNAMBIGUOUSLY

* AVOID A RADIAL COORD. GAUGE FIXING TO GET A MORE TRACTABLE HAMILTONIAN.

1+1 THEORY FOR METRICS

$$ds^2 = -\left(N^2 - (N^r)^2\right)dt^2 + 2N^r\,dr\,dt + \Lambda^2 dr^2 + R^2 d\Omega^2 .$$

WITH

$$T_{ab} = \partial_a \phi\, \partial_b \phi - \frac{1}{2} g_{ab} (\partial \phi)^2$$

1+1 ADM VARIABLES

$$\underbrace{(\Lambda, P_\Lambda) \quad (R, P_R)}_{\text{METRIC \& EXT. CURV.}} \qquad \underbrace{(\varphi, P_\varphi)}_{\text{MATTER}}$$

* TIME GAUGE FIXING ⟶

$$(R, P_R) \quad (\varphi, P_\varphi)$$

$$N\,(N^r) \qquad P_\Lambda(R, P_R, \varphi, P_\varphi)$$

* REDUCED ACTION :

$$S_R = \int dr\,dt \left[P_\varphi \dot{\varphi} + P_R \dot{R} - N^r \left(P_\Lambda' + P_R R' + P_\varphi \varphi' \right) \right]$$

$$+ \int dt \left(P_\Lambda N^r \right) .$$

CLASSICAL EVOLUTION :

EQNS $\quad \dot{\varphi} = \{\varphi, \mathcal{H}(N^r)\}$ etc.

CONTAIN TWO PIECES :

$$\dot{\varphi} = \underbrace{\text{EVOLUTION IN TIME}}_{\text{GENERATED BY}} + \mathcal{L}_{N^r}\varphi \,.$$

GENERATED BY

$$H = \int dr\, (N^r)' P_\Lambda \neq 0$$

WE WOULD LIKE $H \longrightarrow \hat{H}$ ....

BUT

$$P_\Lambda = P_R R + \sqrt{(P_R R)^2 - 16 R^4 \left(H_\varphi - {}^{(3)}\!R\right)} \quad !?$$

$$\left[\begin{array}{l} H_\varphi = \dfrac{P_\varphi^{\,2}}{2R^2} + \dfrac{R^2}{2}\varphi'^2 \\[2ex] {}^{(3)}\!R(R, R', R'') = \text{Ricci scalar of } q_{ab}\,. \end{array}\right]$$

A CONSTRUCTION OF $\hat{H}$ BY LINEARISATION OF $\sqrt{}$:

* $$\widehat{\sqrt{}} = v_1 \hat{A} + v_2 \hat{B} + v_3 \hat{C} + v_4 \hat{D}$$

($A, \ldots D$ ARE THE 4 PIECES IN ROOT)

v_i ARE VECTORS IN A SUBSPACE OF 4x4 MATRICES SATISFYING

$$v_i \cdot v_j = \eta_{ij} I = \mathrm{diag}(1, -1, -1, 1) I$$

* v_i ARE REAL MATRICES.

* QUANTUM STATES ARE REAL SPINORS:

EG. $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} | \underbrace{a_1 \cdots a_k}_{R\text{ excitations}} ; \underbrace{b_1 \cdots b_k}_{\phi\text{ excitations}} \rangle$

(FROM OLIVER'S TALK)

WITH $\hat{H}$ DEFINED WE HAVE

$$i \frac{\partial}{\partial t} |\psi\rangle = \hat{H}(N^\gamma) |\psi\rangle$$

AND

$$\hat{C}(N^\gamma) |\psi\rangle = 0 .$$

* WE CAN TRACK EVOLUTION (IN GAUGE FIXED TIME) WITHOUT SOLVING THE SECOND EQN, FOR GIVEN N^γ (WHICH SATISFIES ASYPT. CONDITIONS).

cf YM theory:

$$i \frac{\partial}{\partial t} |\psi\rangle = \left[\hat{H}_{YM}(E,A) + \Lambda \cdot \hat{G} \right] |\psi\rangle$$

$$\hat{G} |\psi\rangle = 0$$

--- EVOL OF $|\psi\rangle$ POSSIBLE WITHOUT SOLVING $\hat{G}|\psi\rangle = 0$ FOR PRESCRIBED Λ.

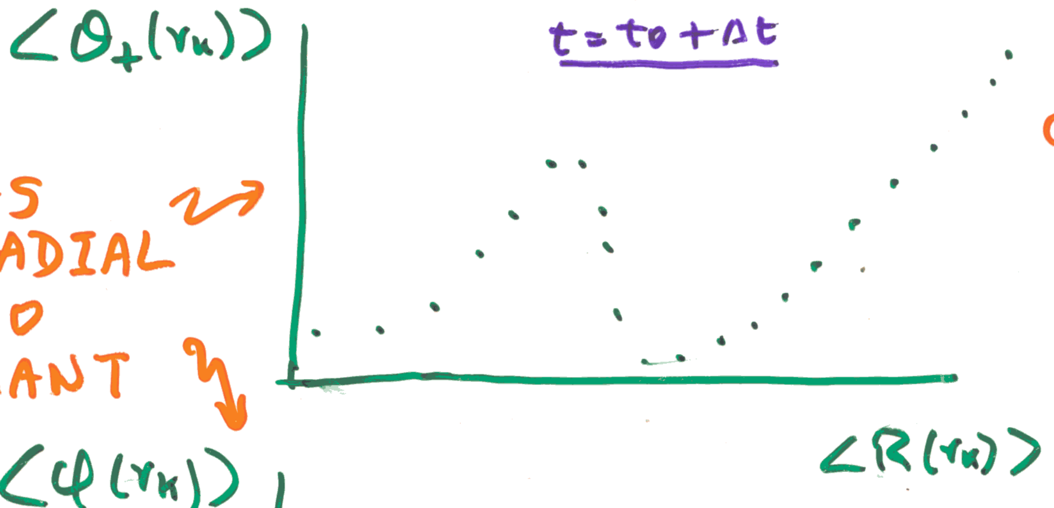
ALGORITHM FOR COLLAPSE EVOL:

1. PICK INITIAL STATE

$|\psi\rangle_{t_0} =$

2. $|\psi\rangle_{t_0+\Delta t} = \left(I + i\,\Delta t\,\hat{H}\right)|\psi\rangle_{t_0}$

3. COMPUTE:



GRAPHS ARE RADIAL DIFFEO INVARIANT

CAN COMPUTE VARIOUS OTHER VARIABLES AT EACH t

4. GOTO 2

SUMMARY / COMMENTS :

* AN EXPLICIT SETUP FOR STUDYING GRAV. COLLAPSE IN QG IN A FIELD THEORY SETTING

* BY HAND CALCULATION FOR GRAPH OF 3 POINTS : SEE MOVEMENT OF $\langle O_+(v_n)\rangle$, $\langle \Phi(v_n)\rangle$ VS $\langle R(v_n)\rangle$ FROM $t_0 \to t_0 + \Delta t$.

* REALISTIC EVOLUTION REQUIRES NUMERICAL WORK.
  EASIER THAN CORRESPONDING CLASSICAL COLLAPSE IMPLEMENTATION.
  IN PROGRESS (. . . . LOOPS 06)

* OTHER GAUGES
* PARTIAL OBSERVABLES
* GAUGE FIXED SPIN NETWORK EVOL
* BOSONISATION
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