FRIDAY 10-15 AM!

"DUALIZATION OF GRAVITY"

B. JULIA

ABSTRACT

1) GRAVITY IS $\mathcal{N}=0$ SUPERGRAVITY

2) IT POSSESSES U-DUALITY
EHLERS - GEROCH - ...

3) IT ADMITS "V-DUALITY": $F = *\tilde{F}$

4) WE DEFINE Λ-INSTANTONS
NAMELY GRAVITATIONAL INSTANTONS FOR A FIXED VALUE OF $\Lambda_{\text{cosmo.}}$

5) Λ-GRAVITY IS SELF-DUAL NEAR DE SITTER SPACE

LOOPS 05
AEI - GOLM

I/ U-V REVIEW

PHILOSOPHY PAGE

II/ $D=4$ $\mathcal{N}=0$ $\Lambda \neq 0$

DUALIZING GRAVITY

A Strategy from the 60's.

FEYNMAN
DE WITT

current algebras IR

↑

YANG-MILLS
"HIGGS"

HERE

GENERAL RELATIVITY

↑

?

RESULTS

- GAUGE FIXING
- RENORMALIZATION

A Strategy for the 200*

IR (General Relativity (+ Quantum Mechanics

$\boxed{\text{THERE}}$

M/STRING Theory

HERE

SPIN FOAMS

RESULTS

- S_{BH} : Entropy
- UV Improvements
- D_p - Branes
- DUALITIES
  AdS/CFT ...
- (BACKGROUND Independence)

CHALLENGES

- Λ_{cosmo} ?
- VACUUM Selection
- or "Matter Selection"

What do I like about DUALITIES?

1/ Good for Strong Coupling

"S-dualities" $\tilde{g} \simeq \frac{1}{g}$

2/ Exchange Quantum/Classical

$$\tilde{g}_{mag.}\, g_{el.} = \frac{\hbar}{2}$$

$q_{\text{NOETHER}} / \text{TOPOLOGY } \tilde{q}$

3/ Good for $\overbrace{\text{Finiteness}}^{}$ quantum (FERMIONS?)

4/ Good for Fermions!

U-Dualities . D=4

	7	6	5	4	3	2	1
$\mathcal{N}$	32	24	20	16	12	8	4 (0)
g_{MN}	1	1	1	1	1	1	1
ψ_M^n	32	24	20	16	12	8	4
A_M	28	16	10	6	3	1	0
χ	"M-theory"					EINSTEIN + MAXWELL	EINSTEIN
φ	70	30	10	2	0	0	0
U	E_7	$SO^*(12)$	$SU(5,1)$	$SU(4) \times SU(1,1)$	$U(3)$	$U(2)$	$U(1)$
KU	$SU(8)$	$U(6)$	$U(5)$	$U(4)$	$U(3)$	$U(2)$	$U(1)$

$$Q_{n=1 \ldots \mathcal{N}}$$

$$=1 \rightarrow 4N \qquad D=4$$

$$\{Q, Q\} = P + c\mathbb{1}$$

AFTER COMPACTIFICATION $\mathcal{M}_3 \times S^1_\theta$

AND REDUCTION

(TRUNCATION) | θ FREE SECTOR

U	E_8	E_7	E_6	$SO(8,2)$	$SU(4,1)$	$SU(2,1) \times SU(2)$	$SU(1,1) \times U(1)$
KU	$SO(16)$	$SO(12) \times SO(3)$	$SO(10) \times U(1)$	$SO(8) \times U(1)$	$SO(6) \times U(1)$	$S(U(2) \times U(1)) \times SU(2)$	$U(1)^2$

CHITRE-KINNERSLEY

EHLERS

$U = SU(2,1)$ ON STATIONARY ELECTROVACS

3D SIGMA MODELS RECALL THEIR MAXIMAL D_{max}.

$$\mathcal{L}_3 = \mathrm{Tr}\left(g^{-1}Dg \wedge g^{-1} * Dg\right) + e R$$

$g \in U$

$d_\mu - \omega_\mu - h_\mu \in \mathrm{Lie}(K_U)$

LORENTZ

D \ N	32
11	.
10	$\mathbb{R}$, $A_1/U(1)$
9	$A_1 \times \mathbb{R}$ ...
8	$A_2 \times A_1$ ...
7	A_4 ...
6	$SO(5,5)$ ...
5	$E_6/USp(8)$
✓ 4	$E_7/SU(8)$
✓ 3	$E_8/SO(16)$
2	E_9/KE_9
1	E_{10}
(0)	?

$A_n = SL(n+1, \mathbb{R})$

$E_9 = E_8^{(1)} = E_8^+$

$\sim \mathrm{Maps}(S^1 \to E_8)$

"$\mathcal{N}$"$=8$ SUGRA $\neq$ SPLIT FORMS:

$D=5$		$SL(2,\mathbb{R})$
$D=4$	$U(2)/U(2)$	$GL(2,\mathbb{R})$ } $SL(3,\mathbb{R})$
$D=3$	$SU(2,1)/S(U(2)\times U(1))$	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$
$D=2$	A_2^{+}	$\vdots$
$D=1$	A_2^{++}	

ELECTROVACS

D	$n=8$	$n=7$	$n=6$	$n=5$	$n=4$	$n=3$	$n=2$	$n=1$
11	$+$							
10	R, A_1	$+$						
9	$R x A_1$	R						
8	$A_1 x A_2$	$R x A_1, A_2$	A_1					
7	E_4	$R \times A_2$	$R \times A_1$	R	$+$			
6	E_5	$A_1 \times A_3$	$R \times A_1^2$	R^2, A_1^2	R			
5	E_6	A_5	A_2^2	$R \times A_1^2$	$R \times A_1$	A_1		
4	E_7	D_6	A_5	$A_1 \times A_3$	$R \times A_2$	$R x A_1, A_2$	R	$+$
3	E_8	E_7	E_6	E_5	E_4	$A_1 x A_2$	$R x A_1$	R, A_1

Table : Disintegration (Oxidations) for split E_n Cosets

C J L P 1999

H J P 2002

CP2 blown up in (11-D) points in general position for n=8 or more generally with A(8-n) singularity

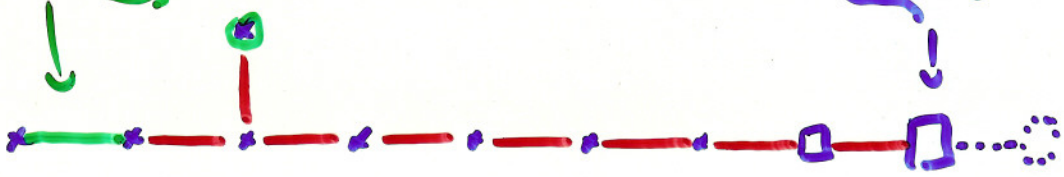
$\mathcal{N} = 32$

$SL(3) \times SL(2)$

$(D = 8)$

Weyl

E_{10} $(D = 1)$



E_{11} ?

$\mathcal{N} = 4, 0$

"Arithmetic"

A_1^{++} $(D = 1)$ BKL. Chaos.

A_1^{+++} ?

$SL(2)$

$(D = 3)$

A_1^{+} GEROCH $+ \dots$

$(D = 2)$

V- Dualities

SCALAR MATTER $\mathcal{M}_D \to U/K_U$

The non-compact U group acts separately on itself : (0- FORMS)

on 1- Forms (+ DOUBLED IF $D=4$)

etc...

FOR $D \geq 3$ TORIC THEORIES

ARE ACTUALLY DEMOCRATIC

$$g = e^{\varphi^i(x)\, T_i} \quad \to \quad \boxed{\mathcal{V}_D = e^{\varphi^i . T_i + \underset{\text{1-FORMS}}{\underset{\uparrow}{A}} . Q \ldots + \underset{\text{D-2 FORMS}}{\underset{\uparrow}{\tilde{\varphi}}} . \tilde{T}}}$$

ALL p-FORMS (AND DUALS)
HODGE

SATISFY

$$\boxed{\mathcal{F} = * \tilde{\mathcal{F}}}$$

WITH

$$\mathcal{F} = \mathcal{V}^{-1} d\mathcal{V}$$

$*$: HODGE DUALIZATION ON FORMS

$\sim$ MEANS $T \to \tilde{T}$

$\tilde{T} \to \pm T = (\tilde{\tilde{T}})$

$$(\tilde{*})^2 = +1$$

NON LINEARITIES IN (SUPER) ALGEBRA : M-CASE:

$$\{Q_3, Q_3\}_+ = \kappa \tilde{Q}_3 \equiv \kappa Q_6$$

NOW:

2 GRAVITON HELICITIES

$\downarrow$

2 SCALARS ($D=3$)

IN GENERAL ALL DEGREES OF FREEDOM END UP AS 0-FORMS i.e. INSIDE U CV

What about $D \geq 4$ GRAVITY?

$$\underline{R = * \tilde{R}}$$

$$R_{MNPQ} = \frac{1}{4} \varepsilon_{MNRS} R^{RSTU} \varepsilon_{TUPQ}$$

EVEN WITH LORENTZ $(+--\cdot)$

($\Leftrightarrow$ EINSTEIN SPACE $R_{MN} \propto g_{MN}$)

V DUALITY

BY HAND

+ CREMMER LU POPE

hep-th 9909099 (+ 2)

AND DEL PEZZOS

+ P. HENRY-LABORDÈRE AND L. PAULOT hep-th (0203070 / 0212346

+ 1 WITH Y. DOLIVET + 3 hep-th 05..YYY

Philosophical remarks

What is?

What is?		
N	0	KEEP OPTIMAL GENERALITY
$D-4$	0	
Λ	0?	$\neq 0$
γ	? θ TYPE	MORE WORK
$\hbar$	$\neq 0$	N

APOLOGIES TO

(DAMOUR - HENNEAUX - NICOLAI
WEST ....

HYPERBOLIC KAC-MOODY ARE THERE : in GR.

A TRIP

GENERAL RELATIVITY

$\color{red}{U}$: MAGIC TRIANGLE $(p=-1)$

STRINGS

?

$\color{red}{V}$: D_p- BRANES , $A_{M_1 \dots M_{p+1}}$

$(p \geq -1)$

$T_A \leftrightarrow \tilde{T}_A$

$$\mathcal{F} = * \tilde{\mathcal{F}}$$

HODGE $\left(\frac{1}{2} \varepsilon_{MNPQ} \mathcal{F}^{PQ} \right)$

$\mathcal{R} = * \tilde{\mathcal{R}}$? Λ : FLOATING

$$Z^{\Lambda}_{MNPQ} \equiv R_{MNPQ} - \frac{\Lambda}{3} \left(g_{MP} g_{NQ} - g_{MQ} g_{NP} \right)$$

IIa

$\left(Z^{\Lambda} = * Z^{\Lambda} , \dots \right)$ Λ FIXED

IIb

$h_{ij}, \pi^{ij} \longrightarrow \varepsilon_{(i}^{\ \cdot\cdot} \partial_{\cdot} \phi_{\cdot j)} \ , \ \varepsilon^{i \cdot\cdot} \varepsilon^{j \cdot\cdot} \partial_{\cdot} \partial_{\cdot} P_{(\cdot\cdot)}$

$\mathcal{H} = 0$ $\mathcal{H}^i = 0$

II a) Λ- INSTANTONS

$$R_{\underline{MN}\,\underline{PQ}}$$

$\frac{1}{2}\,\epsilon_{--}$

NOTATION

ϵ $\ast$ — SPACETIME (HODGE) DUALIZATION

ϵ $\sim$ — (SOMETIMES ON LORENTZ INDICES)

(OLD) PHYSICAL GRAVITATIONAL INSTANTON EQUATION = 0-INSTANTON

$\Lambda = 0$

$$\underline{R = \ast R}$$

$\Leftrightarrow \quad R^A{}_B = \ast R^A{}_B$ LORENTZ CURVATURE

$\Rightarrow \begin{cases} R_{MN} = 0 \\ \text{RICCI FLAT} \end{cases}$

BPS $\left|\mathrm{Tr}\int R \wedge \ast R\right| \geq \left|\mathrm{Tr}\int R \wedge R\right| \doteq |\tau|$

LET US NOT CALL EINSTEIN SPACES INSTANTONS !

YET $\int \mathrm{Tr}(R \wedge * R) \geq \int \mathrm{Tr}(R \wedge \tilde{R}) \sim \chi_{EULER}$

MATHEMATICIANS CALL "SELF-DUAL" SPACES WITH SELF-DUAL WEYL TENSOR :

$$\underline{C = * C}$$

S^4, $\mathbb{C}P^2$ ... TWISTOR SPACES

D=4
++++

$$R \quad \begin{array}{c c} & \begin{array}{cc} + & - \end{array} \\ \begin{array}{c} + \\ - \end{array} & \begin{pmatrix} C_{+}^{+}, r & R^0 \\ R^0 & C_{-}^{-}, r \end{pmatrix} \end{array}$$

r: SCALAR CURVATURE

R^0: TRACE FREE RICCI $= 0 \Leftrightarrow$ E.Sp.

Λ COSMOLOGICAL CONSTANT $\gtrless 0$ $\neq$

define

$$Z^{\Lambda}_{MNPQ} := R_{MNPQ} - \frac{\Lambda}{3}\left(g_{MP}g_{NQ} - g_{MQ}g_{NP}\right)$$

Λ- INSTANTON EQUATION

$$\underline{Z = \pm * Z} \qquad (\Leftrightarrow Z = \pm \tilde{Z})$$

TORSION $= 0$
FOR NOW ...

$$\Rightarrow \quad R_{MN} = \Lambda g_{MN}$$

RICCI

$$\Rightarrow \quad C = \pm * C$$

TWISTOR SPACE

WHAT DOES THE BPS CONDITION(S) TELL US ?

$$\mathrm{Tr}\int (Z_\Lambda * Z) \geq \left| \mathrm{Tr}\int Z_\Lambda Z \right| \equiv |\tau|$$

$$\underbrace{\mathrm{Tr}\int (Z_\Lambda * Z)}_{= S_\Lambda^{(2)}} \geq \underbrace{\mathrm{Tr}\int Z_\Lambda \tilde{Z}}_{= 32\pi^2 \chi_E - \frac{4}{3}\Lambda S_\Lambda = S_\Lambda^{(3)}}$$

$$S_\Lambda = \int (R - 2\Lambda)\sqrt{g}\, d^4x$$

MAC DOWELL MANSOURI

$$\begin{cases} \Lambda S_\Lambda \leq 24\pi^2 \chi_E \\ \Lambda S_\Lambda \geq 24\pi^2 \chi_E - \frac{3}{4}\int \mathrm{Tr}(Z_\Lambda * Z) \end{cases}$$

WE ARE LED BACK TO D=5 (/ SO(5) ?)

AND TO 1977 ! ...

II b/ NEAR de Sitter

DUALITY.

- J. LEVIE
- S. RAY

0507262 (hep-th)

$\Lambda = 0$ HENNEAUX + TEITELBOIM

gr-qc/0408101

$$SO(2)\begin{cases} \mathcal{E}_{mn} = R_{0m0n} \\ B_{mn} = \frac{1}{2}\varepsilon_{npq} R_{0m}{}^{pq} \end{cases}$$

SOLVE $\partial^m \partial^n h_{mn} = \Delta h$

$\pi^{mn}{}_{,n} = 0$ $\quad$ ($m, n \ldots = 1 \to 3$

$$-L_\Lambda = \pi^{ij}\dot{g}_{ij} + N\, g_{(3)}^{1/2}\left[R^{(3)} - 2\Lambda\right]$$
$$+ N\, g_{(3)}^{-1/2}\left[\pi^2/2 - g_{ik}\, g_{jl}\, \pi^{ij}\pi^{kl}\right]$$
$$+ 2N_i\left[\partial_j \pi^{ij} + \Gamma^{(3)\,i}{}_{jl}\, \pi^{jl}\right]$$

WITH

$N = (-g_{00})^{1/2}$ $\quad N_i = g_{0i}$

$g_{(3)} \equiv \det g_{ij}$ $\quad \pi \equiv g_{ij}\, \pi^{ij}$

π^{ij} = MOMENTUM (DENSITY)
"DUAL" TO g_{ij}

NOW

a) LINEARIZE $\quad g = \bar{g} + h \ldots$

$N = 1 + n \quad , \quad g_{ij} = f^2(t)\,\delta_{ij} + h_{ij}$

$\pi^{ij} = \bar{\pi}^{ij} + p^{ij}$

$$d\bar{s}^2 = -dt^2 + f^2(t)\,\delta_{ij}\,dx^i dx^j$$

$$f = e^{\sqrt{\Lambda/3}\,t} \qquad K \equiv \sqrt{\Lambda/3}$$

b) SOLVE CONSTRAINTS

- VECTOR CONSTRAINT

$$p^{ij} = K f^{-1}\left(2h^{ij} - \delta^{ij} h\right) + f^{-1}\,\varepsilon^{ik\ell}\,\varepsilon^{jmn}\,\partial_\ell \partial_m P_{kn}$$

- SCALAR CONSTRAINT

$$h_{ij} = f\left(\varepsilon_{ik\ell}\,\partial^k \phi^\ell_j + i \leftrightarrow j\right) + \partial_i u_j + \partial_j u_i - 2K f^2 P_{ij}$$

c) INVARIANCES + PRE-INVARIANCES

$$\delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

$$\delta u_i = \xi_i \; ..$$

$$\delta \Gamma_{ij} = \partial_i \alpha_j + \partial_j \alpha_i + 2\kappa f^{-1}\left(\partial_i \xi_j + \partial_j \xi_i + \cdots\right)$$

$$\delta \phi_{ij} = \partial_i \beta_j + \partial_j \beta_i$$

(MORE SYMMETRIES YET)

d) DUALITY SYMMETRY

$$\Sigma_{ij} \equiv Z_{i0j0}$$

$$B_{ij} \equiv \tilde{Z}_{i0j0}$$

ON SHELL (= USING E.O.M.)

$$\begin{cases} \delta \Sigma = B \\ \delta B = -\Sigma \end{cases}$$

$$\begin{cases} \delta \phi = P \\ \delta P = -\phi \end{cases}$$

OFF SHELL

$$-fL = \Delta\phi\,\Delta\phi + \Delta P\,\Delta P + 2\,\varepsilon_{ijk}\,\partial_\ell \phi^{k\ell}\,\partial^j \partial_m P^{im} + \cdots$$

PROSPECTS

1) DISPENSE WITH ASYMPTOTIC FLATNESS

NUT Charge

2) INTEGRABILITY ?

3) MORE Λ- INSTANTONS

4) NON-LINEAR "DUALITY" FOR GRAVITY ITSELF

5) QUANTUM MEASURE SHOULD PRESERVE AS MUCH DUALITY AS POSSIBLE