LQG Black Holes :

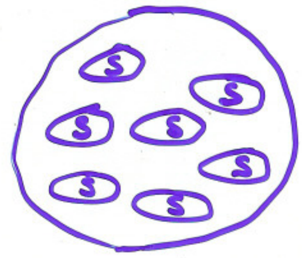
Entropy, Entanglement, Evaporation and a link with Quantum Information

gr-qc/0508085, collaborat° with Daniel Terno

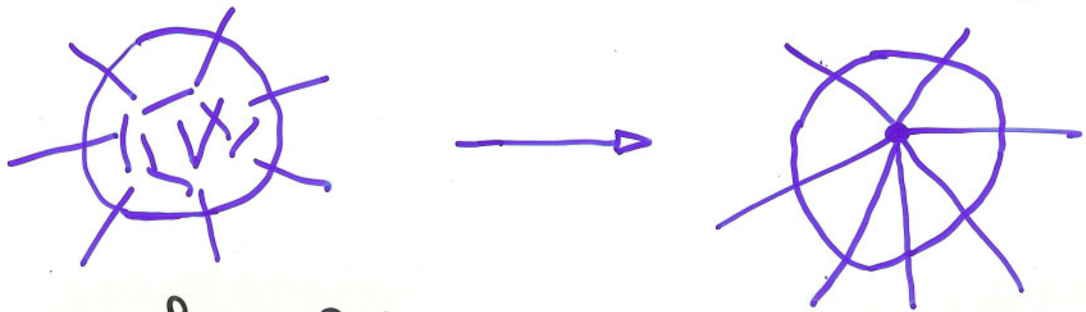
Perimeter Institute

A simple model for the horizon

⇝ a closed surface made of
n patches of a fixed spin s



⇝ Black Hole : we "ignore" the inside geometry



complex graph(s)
= full/true quantum state

a single vertex
= coarse-grained state
= surface state

⇒ BH states for the external observer
= horizon states
= intertwiners $V^s \otimes V^s \otimes \ldots \otimes V^s \longrightarrow \mathbb{C}$ SU(2)-inv ⚠

⇔ Entropy will count the nb of intertwiners

⇝ fundamental model for spin $s=\frac{1}{2}$ ⇝ the "qubit BH"

Why this model?

⇝ spin s = unit of area of the observer measuring the horizon. → a "ruler"

highest resolut° for smallest unit $s = \frac{1}{2}$

⇝ so not to overcount intertwiner states ...

$$\mathrm{Inv}\Big(\underbrace{\tfrac{1}{2}\otimes\tfrac{1}{2}\otimes\ldots\otimes\tfrac{1}{2}}_{2(n-1)}\otimes 1\Big) \leftrightarrow \mathrm{Inv}\Big(\underbrace{\tfrac{1}{2}\otimes\ldots\otimes\tfrac{1}{2}\otimes\tfrac{1}{2}\otimes\tfrac{1}{2}}_{2n}\Big)$$

⇝ $\mathrm{Inv}((V^{s})^{\otimes n}) = \sigma_n^{(s)}$ is a rep of permutat° group $\mathfrak{S}_n$

$\leftrightarrow$ "discrete diffeos" on the horizon.

for $s = \frac{1}{2}$, $\sigma_n^{(1/2)}$ is irreducible $\overset{?}{\leftrightarrow}$ Fundamental!

⇝ looking for states allowing an approximate $SO(3)$ act° which converges to std act° in classical limit $n \to \infty$

(Dreyer, Markopoulou, Smolin)

→ same spin s for all patches

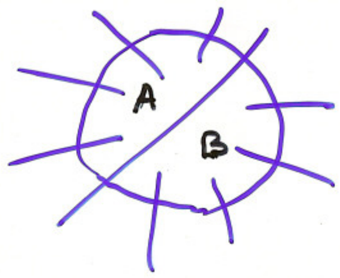
What do we want?

⇝ **Entropy**: look @ state "we don't know the intertwiner"

$\mathcal{H} = \mathrm{Inv}\left((V^{s})^{\otimes n}\right)$ $\quad \rho = \frac{\mathbb{1}}{\dim \mathcal{H}}$ density matrix

$\Rightarrow S = -\mathrm{Tr}\,\rho \ln(\rho) = \ln(\dim \mathcal{H}) \underset{\uparrow \text{WORK}}{=} \alpha(s)\, n - \underbrace{\frac{3}{2} \ln n}_{\text{ind}^t \text{ of } s!} + \dots$

⇝ **Correlat° / Entanglem^t**: responsible for log correct°

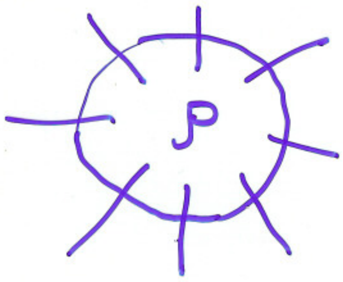


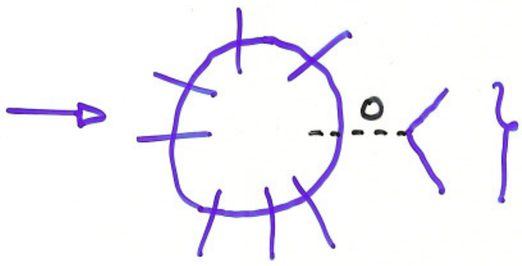
$\mathrm{Cor}(A|B)$, $\mathrm{Ent}(A|B)$?

for size $\frac{n}{2}$: $\mathrm{Cor}(A|B) \sim \frac{3}{2} \ln n$

$\mathrm{Ent}(A|B) \sim \frac{1}{2} \ln n$

⇝ **Evaporation**: Prob(evap of block A) in terms of Ent (A | rest)





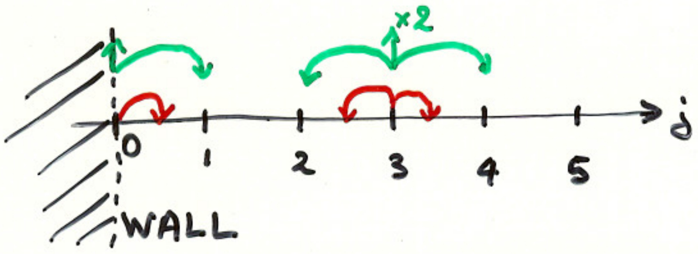
couple of links detached/ing from the horizon

not a dynamical process ... yet ...
but purely quantum feature !

Spin ½ BH model : Entropy from Random Walk

⇝ decomposit° $\underbrace{V^{\frac12}\otimes \cdots \otimes V^{\frac12}}_{2n \text{ times}} = \bigoplus_{j=0}^{n} d_j^{(n)}\, V^j$

Let's compute the degeneracies $d_j^{(n)}$ iteratively :



$\otimes(V^{\frac12}\otimes V^{\frac12}) = \otimes(V^0 \oplus V^1)$

$\otimes V^{\frac12}$

$d_j^{(n)}$ = nb of returns of RW with wall to spot j after n steps

$= RW_n(j) - RW_n(j+1)$!!

⇝ explicitly : $d_j^{(n)} = C_{2n}^{j} - C_{2n}^{j+1} = \frac{2j+1}{n+j+1}\, C_{2n}^{n+j}$

→ $\dim\left(\mathrm{Inv}(V^{\frac12})^{\otimes 2n}\right) = d_0^{(n)} = \frac{1}{n+1} C_{2n}^{n} \sim \frac{2^{2n}}{n\sqrt{n}}$ ←

$\Rightarrow S = \ln d_0^{(n)} \sim 2n\log 2 \boxed{-\frac{3}{2}} \log n - \frac12 \log \pi + \ldots$

↪ intuit° : $RW_n \sim \frac{2^{2n}}{\sqrt{n}} \Rightarrow \Delta RW_n \simeq \frac{2^{2n}}{n\sqrt{n}} \sim \frac{1}{n} RW_n$

→ $d_j^{(n)}$ max for $j_{max} \sim \sqrt{\frac{n}{2}}$

$d_{\sqrt{\frac{n}{2}}}^{(n)} \sim \frac{2^{2n}}{n}$ ←

About Entanglement ...

⇝ example $|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\underset{A}{\uparrow}\underset{B}{\downarrow}\rangle - |\underset{A}{\downarrow}\underset{B}{\uparrow}\rangle\right)$ $\qquad V^0 \hookrightarrow V^{1/2} \otimes V^{1/2}$

$\rho_A = \mathrm{Tr}_B\, |\Psi_0\rangle\langle\Psi_0| = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ = totally mixed state

$\mathrm{Cor}(A|B) = \Delta S = S(\rho_A) + S(\rho_B) - S(\rho) = 2 \ln 2$

$\hookrightarrow$ 2 "bits" of correlation

$\mathrm{Ent}(A|B) = S(\rho_A) = \ln 2 = \frac{1}{2}\,\mathrm{Cor}(A|B)$

$\hookrightarrow$ 1 bit of entanglem^t ⇝ q-cor, can be used in teleportat$^°$ protocols

⇝ in general: pure state $\rho = |\Psi\rangle\langle\Psi|$ $\qquad \mathrm{Ent} = S(\rho_A^{red})$

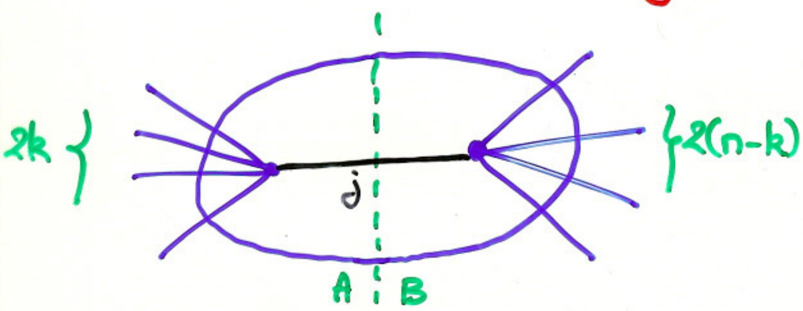
mixed state $\rho = \sum_i \omega_i |\Psi_i\rangle\langle\Psi_i|$ $\qquad \underline{\underline{\mathrm{Ent} = \min_{\{|\Psi_i\rangle\}} \sum_i \omega_i\, \mathrm{Ent}(\Psi_i)}}$

⇝ for mixed states, $\exists$ several defs of Ent related to their practical / theoretical use.

+ it's hard calculation (min ...)

BUT in our case, all defs of Ent coincide and the actual Ent is computable!

Computing the Entanglement



$$(V^{\frac{1}{2}})^{\otimes 2n} = (V^{\frac{1}{2}})^{\otimes 2k} \otimes (V^{\frac{1}{2}})^{\otimes 2(n-k)}$$

$$\Rightarrow (V^{\frac{1}{2}})^{\otimes 2n} = \left(\bigoplus_{j_A=0}^{k} d^{(k)}_{j_A} V^{j_A}\right) \otimes \left(\bigoplus_{j_B=0}^{n-k} d^{(n-k)}_{j_B} V^{j_B}\right)$$

$$= \bigoplus_{j_A, j_B} d^{(k)}_{j_A} d^{(n-k)}_{j_B} (V^{j_A} \otimes V^{j_B})$$

invariant sector (spin 0) given by $j_A = j_B$: $\mathrm{Inv}(V^{\frac{1}{2}})^{\otimes 2n} = \bigoplus_j d^A_j d^B_j \; V^0_{(j,j)}$

$\Rightarrow$ intertwiner states $|j, \underbrace{a_j, b_j}\rangle$

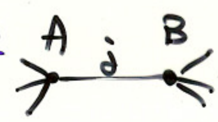
label a basis in degeneracy space d^A_j, d^B_j

... a ¿little? work ...

$$\mathrm{Ent}(A|B) = \frac{1}{N} \sum_j d^A_j d^B_j \ln(2j+1)$$

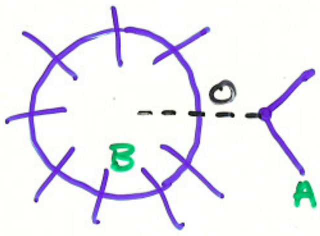
with $N = d^{(n)}_0 = \dim \mathrm{Inv}(V^{\frac{1}{2}})^{\otimes 2n} = \sum_j d^A_j d^B_j$

is the dim of the full intertwiner space

and $d^A_j d^B_j$ is the dim of intertwiners of the type 

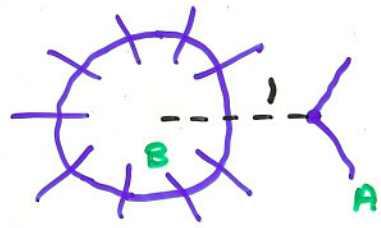
Evaporation from (non-) Entanglement

⇝ focus on one pair of qubits and look at:



Ent = 0

versus



Ent > 0

Quantified by unentangled fract° f:

$$\underbrace{f \equiv \mathrm{Tr}\left(\mathrm{Proj}_{B \underset{0}{-} A} \cdot \rho\right)}_{\text{Probability of "measuring" A detached from horizon!}} = \frac{\dim \mathcal{H}_{B \underset{0}{-} A}}{N} = \frac{d_0^A d_0^B}{N}$$

⇝ proba $\xrightarrow{\text{time scale } \tau}$ rate

semi-classically at large n: $\quad f \sim \frac{1}{4} + \frac{3}{8n}$

$\tau \sim \text{Mass} \propto \sqrt{\text{Area}} \propto \sqrt{n}$

$$\Rightarrow \frac{d\,\text{Area}}{dt} \propto \frac{dn}{dt} \propto -\frac{f}{\tau} \propto -\underbrace{\frac{1}{4\sqrt{n}}}_{\text{Hawking evaporat° term}} - \underbrace{\frac{3}{8n\sqrt{n}}}_{\text{Correct°}}$$

☺

Recall: $\quad \frac{dM}{dt} \propto -\frac{1}{M^2} \Leftrightarrow \frac{dn}{dt} \propto -\frac{1}{\sqrt{n}}$

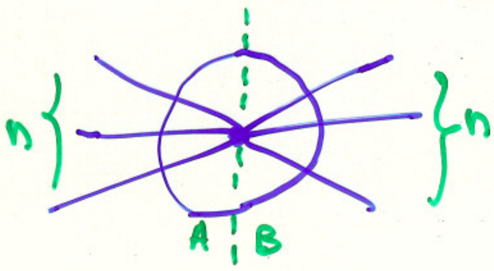
Remark:

Can't avoid "evaporat°" ...

without significantly decreasing

the entropy!

Cutting the horizon in halves



$$\mathrm{Ent}(n|n) \sim \underline{\frac{1}{2}\ln n}$$

$$\mathrm{Cor}(n|n) = S(\rho_n) + S(\rho_n) - S(\rho) \sim \underline{\frac{3}{2}\ln n} \sim 3\,\mathrm{Ent}(n|n)$$

↳ Responsible for the log correct° to the entropy law!

→ why a factor 3?

a pure entangled state gives Cor = 2 Ent,
here we have more classical correlations ...

It can be translated in the language of
quantum information!

Mapping (L)QG (BH) to QI

Soraya ⟶ Tamara

↑ ref frame

$\underbrace{\uparrow\downarrow\uparrow\uparrow}_{n \text{ qubits}}$ ⇝ measures the qubit ...
in which ref frame ?!

↪ without shared ref frame: only transmittable data is ...

$\mathrm{Inv}\,(V^{\frac{1}{2}})^{\otimes n}$ ⟷ Intertwiner space

↪ cryptographic power of ref frames (Bartlett, Rudolph, Spekkens, Terno)

↗ Soraya ⟶ Tamara ↗

↑↓↗↙↘↑ ⇝

Marc tries to eavesdrop, but don't know the ref frame!

the only data Marc can access is the one encoded in $\mathrm{Inv}((V^{\frac{1}{2}})^{\otimes n})$...

So S & T just put junk in $\mathrm{Inv}(V^{\frac{1}{2}})^{\otimes n}$ ...

... some work later ...

factor 3 :)

classical channel capacity ⟷ Cor(n|n) of the LQG BH

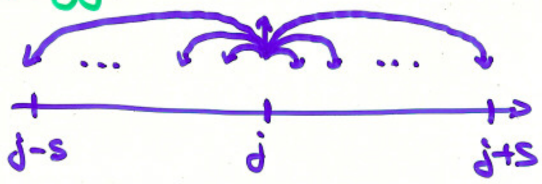
quantum channel capacity ⟷ Ent(n|n)

Higher spin models

with arbitrary spin s ... it all works the same!

⇝ Random Walk analogy:

$\otimes V^s \quad \longleftrightarrow$



$\exists$ integral formula for $RW_n^{(s)}(j)$ $\Rightarrow$ same asymptotics $S \sim n \ln(2s+1) \boxed{-\frac{3}{2}} \ln n + \dots$

$\Rightarrow j_{max} \propto \sqrt{n}$ for max degeneracy $d^{(s)}_{max} \sim \frac{(2s+1)^n}{n}$

⇝ Entanglem^t calculat° depends solely on degeneracy coeff and leads to the same behavior.

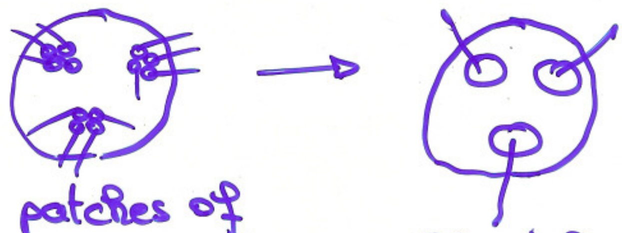
Conclusion:

- factor $-3/2$ is universal
  ind^t of area unit, inv under coarse-graining
- asymptotical behavior of entanglem^t/correlat° is ind^t of unit s
  $\Rightarrow$ link entanglem^t $\longleftrightarrow$ evaporation seems universal.

Outlook

↪ Renormalisat° of Geometry

ex: Area



$\mathcal{H} = (V^{\frac{1}{2}})^{\otimes k}$

n patches of spin ½ (or s_0)

n/k patches of spin s?

Looking for most probable s : S_{max} or $\langle s \rangle$ or... always $\propto \sqrt{k}$

$\Rightarrow A_{micro} = n\, a_{\frac{1}{2}} > A_{macro} \sim \frac{n}{k} a_{\sqrt{k}} \sim \frac{n}{\sqrt{k}}$ ↓ with k

Volume? (with Tina Giesel)

↪ Interact° Entanglem^t ⟷ Geometry   QI ⇄ QG

looking for an interpretat° of entanglem^t as a not° of distance in a background ind^t theory of geometry. (with Dany Terno, Florian Girelli)

(think of scaling Ent(dist) in spin systems)

↪ Dynamics : a Hamiltonian ?!?

to get a time scale

to understand "semi-classical" state

to derive the evaporat° process.