Master Constraint Operator for Loop Quantum Gravity

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Outline

- Introduction: Backgroud and Idea
- A Self-adjoint Master Constraint Operator
- Discussion and Outlook
- Quantum Gravity at Beijing Normal Univ

1. Introduction

- Connection dynamics of GR

  The Hamiltonian:

$$H_{tot} = \mathcal{G}(\Lambda) + \mathcal{V}(\vec{N}) + \mathcal{H}(N),$$

with the constraints algebra

$$\begin{aligned}
\{\mathcal{G}(\Lambda),\ \mathcal{G}(\Lambda')\} &= \mathcal{G}([\Lambda,\ \Lambda']),\\
\{\mathcal{G}(\Lambda),\ \mathcal{V}(\vec{N})\} &= -\mathcal{G}(\mathcal{L}_{\vec{N}}\Lambda),\\
\{\mathcal{G}(\Lambda),\ \mathcal{H}(N)\} &= 0,\\
\{\mathcal{V}(\vec{N}),\ \mathcal{V}(\vec{N}')\} &= \mathcal{V}([\vec{N},\ \vec{N}']),\\
\{\mathcal{V}(\vec{N}),\ \mathcal{H}(M)\} &= -\mathcal{H}(\mathcal{L}_{\vec{N}}M),\\
\{\mathcal{H}(N),\ \mathcal{H}(M)\} &= -\mathcal{V}((N\partial_b M - M\partial_b N)\frac{\widetilde{P}^a_i \widetilde{P}^{bi}}{|\det q|})\\
&\quad -\mathcal{G}((N\partial_b M - M\partial_b N)A_a\frac{\widetilde{P}^a_i \widetilde{P}^{bi}}{|\det q|})\\
&\quad -(1+\gamma^2)\mathcal{G}(\frac{[\widetilde{P}^a\partial_a N, \widetilde{P}^b\partial_b M]}{|\det q|}).
\end{aligned}$$

Characters of the above Poisson algebra:

- $\star$ The algebra generated by the Gaussian constraints $\mathcal{G}(\Lambda)$ forms not only a subalgebra but also a 2-side ideal in the full constraint algebra.
- $\star$ The subalgebra generated by the diffeomorphism constraints $\mathcal{V}(\vec{N})$ can not form an ideal.
- $\star$ It is not a Lie algebra, because the Poisson bracket between the two scalar (Hamiltonian) constraints $\mathcal{H}(N)$ and $\mathcal{H}(M)$ has structure function depending on dynamical variables even modulo the Gauss constraint.

The last two characters cause much trouble in solving the constraints in loop quantum gravity.

- Hamiltonian constraint operator in LQG

  Although the kinematical Hilbert space $\mathcal{H}_{Kin} := L^2(\overline{\mathcal{A}}, d\mu_{AL})$ and the diffeomorphism invariant Hilbert space $\mathcal{H}_{Diff}$ have been constructed rigorously [Ashtekar el, JMP 36(1995), 6456], the quantum dynamics is still an open issue. Given any cylindrical function $\psi_\alpha \in \mathcal{H}_{Kin}$ and certain state-dependent triangulation $T(\epsilon)$, the dual Hamiltonian constraint operator $\hat{\mathcal{H}}'(N)$ acts on a diffeomorphism invariant state $\Psi_{Diff} \in \mathcal{H}_{Diff}$ as

$$(\hat{\mathcal{H}}'(N)\Psi_{Diff})[\psi_\alpha] = \lim_{\epsilon\to 0} \Psi_{Diff}(\hat{\mathcal{H}}^\epsilon(N)\psi_\alpha),$$

  where the regulated Hamiltonian constraint operator $\hat{\mathcal{H}}^\epsilon(N)$ is densely defined in $\mathcal{H}_{Kin}$ as

$$\hat{\mathcal{H}}^\epsilon(N)\psi_\alpha = (\hat{\mathcal{H}}^\epsilon_E(N) - 2(1+\gamma^2)\hat{\mathcal{T}}^\epsilon(N))\psi_\alpha = \sum_{v\in V(\alpha)} N(v)\hat{\mathcal{H}}^\epsilon_v\psi_\alpha,$$

  here the action of $\hat{\mathcal{H}}^\epsilon_v$ on ψ_α adds edges $e_{ij}(\Delta)$ with $\frac{1}{2}$-representation to the vertex $v(\Delta)$ of α [Thiemann, CQG 15(1998), 839].

Is there any quantum anomaly?

Good evidence:

- ⋆ The action of the dual commutator of two Hamiltonian constraint operators on $\Psi_{Diff} \in \mathcal{H}_{Diff}$

$$([\hat{\mathcal{H}}(N), \hat{\mathcal{H}}(M)])' \Psi_{Diff} = 0$$

- ⋆ The dual commutator between the Hamiltonian constraint operator and finite diffeomorphism transformation operator

$$([\hat{\mathcal{H}}(N), \hat{U}_{\varphi}])' \Psi_{Diff} = \hat{\mathcal{H}}'(\varphi^* N - N) \Psi_{Diff}$$

Several *unsettled* problems:

- $\star$ It is unclear whether the commutator between two Hamiltonian constraint operators resembles the classical Poisson bracket between two Hamiltonian constraints. Hence it is doubtful whether the quantum Hamiltonian constraint produces the correct quantum dynamics with correct classical limit.

- $\star$ The dual Hamiltonian constraint operator does not leave $\mathcal{H}_{Diff}$ invariant. The inner product structure of $\mathcal{H}_{Diff}$ cannot be employed in the construction of physical inner product.

- $\star$ Classically the collection of Hamiltonian constraints do not form a Lie algebra. So one cannot employ group averaging strategy in solving the Hamiltonian constraint quantum mechanically.

  Where is the way out?

- Master constraint program

  Idea: If one could construct an alternative classical constraint algebra, giving the same constraint phase space, which is a Lie algebra and where the subalgebra of diffeomorphism constraints forms an ideal, then the programme of solving constraints would be much improved at a basic level.

  Introduce the master constraint [Thiemann, gr-qc/0305080]:

$$\mathbf{M} := \frac{1}{2}\int_{\Sigma} d^3x \frac{|\widetilde{C}(x)|^2}{\sqrt{|\det q(x)|}},$$

  where $\widetilde{C}(x)$ is the scalar constraint. One then gets the master constraint algebra as a Lie algebra:

$$\begin{aligned}
\{\mathcal{V}(\vec{N}),\ \mathcal{V}(\vec{N}')\} &= \mathcal{V}([\vec{N},\vec{N}']), \\
\{\mathcal{V}(\vec{N}),\ \mathbf{M}\} &= 0, \\
\{\mathbf{M},\ \mathbf{M}\} &= 0,
\end{aligned}$$

  where the subalgebra of diffeomorphism constraints forms an ideal.

  So it is possible to define a corresponding master constraint operator on $\mathcal{H}_{Diff}$.

2. A Self-adjoint Master Constraint Operator

2.1. Define the master constraint operator $\hat{\mathrm{M}}$

- Regularization

  The regularized version of the master constraint

$$\mathbf{M}^{\epsilon} := \frac{1}{2}\int_{\Sigma} d^3y \int_{\Sigma} d^3x \chi_{\epsilon}(x-y)\frac{\widetilde{C}(y)}{\sqrt{V_{U_y^{\epsilon}}}}\frac{\widetilde{C}(x)}{\sqrt{V_{U_x^{\epsilon}}}},$$

  where $\chi_{\epsilon}(x-y)$ is any 1-parameter family of functions such that $\lim_{\epsilon\to 0}\chi_{\epsilon}(x-y)/\epsilon^3 = \delta(x-y)$ and $\chi_{\epsilon}(0)=1$.

  Introducing a partition $\mathcal{P}$ of the 3-manifold Σ into cells C, we have an operator $\hat{H}^{\epsilon}_C$ acting on any cylindrical function $f_{\alpha} \in \mathcal{H}_{Kin}$ via a state-dependent triangulation $T(\epsilon)$ on Σ

$$\hat{H}^{\epsilon}_C \, f_{\alpha} = \sum_{v\in V(\alpha)} \frac{\chi_C(v)}{C^3_{n(v)}} \sum_{v(\Delta)=v} \hat{h}^{\epsilon,\Delta}_v \, f_{\alpha}, \qquad (1)$$

  where $\chi_C(v)$ is the characteristic function of the cell C.

The expression of $\hat{h}_v^{\epsilon,\Delta}$ reads

$$\hat{h}_v^{\epsilon,\Delta} = \frac{16}{3i\hbar\kappa^2\gamma}\epsilon^{ijk}\mathrm{Tr}(\hat{A}(\alpha_{ij}(\Delta))\hat{A}(e_k(\Delta))^{-1}[\hat{A}(e_k(\Delta)), \sqrt{\hat{V}_{U_v^\epsilon}}])$$
$$+2(1+\gamma^2)\frac{4\sqrt{2}}{3i\hbar^3\kappa^4\gamma^3}\epsilon^{ijk}\mathrm{Tr}(\hat{A}(e_i(\Delta))^{-1}[\hat{A}(e_i(\Delta)), \hat{K}^\epsilon]$$
$$\hat{A}(e_j(\Delta))^{-1}[\hat{A}(e_j(\Delta)), \hat{K}^\epsilon]\hat{A}(e_k(\Delta))^{-1}[\hat{A}(e_k(\Delta)), \sqrt{\hat{V}_{U_v^\epsilon}}]),$$

which is similar to the previous regulated Hamiltonian constraint operator. The only difference is that now the volume operator is replaced by its quare-root.

Thus, for each $\epsilon > 0$, $\hat{H}_C^\epsilon$ is a well-defined Yang-Mills gauge invariant and diffeomorphism covariant operator in $\mathcal{H}_{Kin}$.

- Definition

  Define a master constraint operator, $\hat{\mathbf{M}}$, in $\mathcal{H}_{Diff}$ as

$$\hat{\mathbf{M}} := \lim_{\mathcal{P}\to\Sigma;\epsilon,\epsilon'\to 0} \sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}'^{\epsilon\dagger}_C \hat{H}'^{\epsilon'}_C, \tag{2}$$

  where $\hat{H}'^{\epsilon\dagger}_C$ and $\hat{H}'^{\epsilon'}_C$ are well defined by

$$(\hat{H}'^{\epsilon'}_C \Psi)[f_\alpha] := \Psi[\hat{H}^{\epsilon}_C f_\alpha],$$
$$(\hat{H}'^{\epsilon\dagger}_C \Psi)[f_\alpha] := \Psi[\hat{H}^{\epsilon\dagger}_C f_\alpha],$$

  for any cylindrical function $f_\alpha \in Cyl$, and any $\Psi \in Cyl^\star$, here $Cyl^\star$ is the algebraic dual of the set of cylindrical functions Cyl.

  Since the actions of $\hat{H}^{\epsilon}_C$ and $\hat{H}^{\epsilon\dagger}_C$ on any f_α only add finite edges with $\frac{1}{2}$-representations to the graph α, one has $\lim_{\mathcal{P}\to\sigma} \sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}^{\epsilon}_C \hat{H}^{\epsilon'\dagger}_C f_\alpha \in Cyl$, and hence given any $\Psi_{Diff} \in \mathcal{H}_{Diff}$, the value of

$$(\hat{\mathbf{M}}\Psi_{Diff})[f_\alpha] := \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \Psi_{Diff}[\sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}^{\epsilon}_C \hat{H}^{\epsilon'\dagger}_C f_\alpha] \tag{3}$$

  is finite.

For any diffeomorphism transformation φ,

$$\begin{aligned}(\hat{U}'_\varphi \hat{\mathbf{M}} \Psi_{Diff})[f_\alpha] &= \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \Psi_{Diff}[\sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}^\epsilon_C \hat{H}^{\epsilon'\dagger}_C \hat{U}_\varphi f_\alpha] \\ &= \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \Psi_{Diff}[\hat{U}_\varphi \sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}^\epsilon_{\varphi^{-1}(C)} \hat{H}^{\epsilon'\dagger}_{\varphi^{-1}(C)} f_\alpha] \\ &= \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \Psi_{Diff}[\sum_{C\in\mathcal{P}} \frac{1}{2} \hat{H}^\epsilon_C \hat{H}^{\epsilon'\dagger}_C f_\alpha].\end{aligned}$$

Hence $\hat{\mathrm{M}}$ leaves $\mathcal{H}_{Diff}$ invariant

$$(\hat{U}'_\varphi \hat{\mathbf{M}} \Psi_{Diff})[f_\alpha] = (\hat{\mathbf{M}} \Psi_{Diff})[f_\alpha].$$

In conclusion, $\hat{\mathrm{M}}$ is densely defined in $\mathcal{H}_{Diff}$.

2.2. Self-adjointness of $\hat{\mathrm{M}}$

Given two diffeomorphism invariant cylindrical functions $\eta(f_\beta)$ and $\eta(g_\alpha)$ associated with the cylindrical functions f_β and g_α, the matrix element of $\hat{\mathrm{M}}$ is calculated as

$$
\begin{aligned}
& < \eta(f_\beta)|\hat{\mathbf{M}}|\eta(g_\alpha) >_{Diff} \\
= \; & \overline{(\hat{\mathbf{M}}\eta(g_\alpha))[f_\beta]} \\
= \; & \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \sum_{C\in\mathcal{P}} \frac{1}{2}\overline{(\eta(g_\alpha))[\hat{H}^\epsilon_C \hat{H}^{\epsilon'\dagger}_C f_\beta]} \\
= \; & \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \sum_{C\in\mathcal{P}} \frac{1}{2}\frac{1}{n_\alpha} \sum_{\varphi\in Diff/Diff_\alpha} \sum_{\varphi'\in GS_\alpha} \overline{< \hat{U}_\varphi \hat{U}_{\varphi'} g_\alpha | \hat{H}^\epsilon_C \hat{H}^{\epsilon'\dagger}_C f_\beta >_{Kin}} \\
= \; & \lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0} \sum_{C\in\mathcal{P}} \\
& \sum_s \frac{1}{2}\frac{1}{n_\alpha} \sum_{\varphi\in Diff/Diff_\alpha} \sum_{\varphi'\in GS_\alpha} \overline{< \hat{U}_\varphi \hat{U}_{\varphi'} g_\alpha | \hat{H}^\epsilon_C \Pi_s >_{Kin} < \hat{H}^{\epsilon'}_C \Pi_s | f_\beta >_{Kin}}
\end{aligned}
$$

where n_α is the number of the elements of the group, GS_α, of colored graph symmetries of α, $Diff_\alpha$ denotes the subgroup of $Diff$ which maps α to itself, $\gamma(s)$ is the graph associated with the spin-network function Π_s, and the resolution of identity trick is used in the last step.

Split the sum $\sum_s$ into $\sum_{[s]}\sum_{s\in[s]}$, where $[s]$ denotes the diffeomorphism equivalent class associated with s. Since the sum over $[s]$ in the expression is finite, we can exchange $\lim_{\mathcal{P}\to\sigma;\epsilon,\epsilon'\to 0}\sum_{C\in\mathcal{P}}$ and $\sum_{[s]}$, then take the limit $C\to v$,

$$
\begin{aligned}
& <\eta(f_\beta)|\hat{\mathbf{M}}|\eta(g_\alpha)>_{Diff} \\
= & \sum_{[s]}\sum_{v\in V(\gamma(s\in[s]))}\frac{1}{2}\lim_{\epsilon,\epsilon'\to 0}\overline{<\eta(g_\alpha)|\eta(\hat{H}^\epsilon_v\Pi_s)>_{Diff}\sum_{s\in[s]}<\hat{H}^{\epsilon'}_v\Pi_s|f_\beta>_{Kin}} \\
= & \sum_{[s]}\sum_{v\in V(\gamma(s\in[s]))}\frac{1}{2}\lim_{\epsilon,\epsilon'\to 0}\overline{<\eta(g_\alpha)|\eta(\hat{H}^\epsilon_v\Pi_s)>_{Diff}<\eta(\hat{H}^{\epsilon'}_v\Pi_s)|\eta(f_\beta)>_{Diff}} \\
= & \sum_{[s]}\sum_{v\in V(\gamma(s\in[s]))}\frac{1}{2}\overline{(\hat{H}'_v\eta(g_\alpha))[\Pi_{s\in[s]}]}(\hat{H}'_v\eta(f_\beta)[\Pi_{s\in[s]}],
\end{aligned}
$$

where in the first step we use the fact that, given $\gamma(s)$ and $\gamma(s')$ which are different up to a diffeomorphism transformation, there is always a diffeomorphim φ transforming the graph associated with $\hat{H}^\epsilon_v\Pi_s$ $(v\in\gamma(s))$ to that of $\hat{H}^\epsilon_{v'}\Pi_{s'}$ $(v'\in\gamma(s'))$ with $\varphi(v)=v'$, hence $<\eta(g_\alpha)|\eta(\hat{H}^\epsilon_v\Pi_s)>_{Diff}$ is constant for different $s\in[s]$. In the second step, we use the fact that the sums $\sum_{s\in[s]}$ and $\sum_{\gamma(s)\cup a(v)\in[\gamma(s)\cup a(v)]}$, where $a(v)$ is the loop with scale ϵ' added at the vertex v by the operator $\hat{H}^{\epsilon'}_v$, are different up to the diffeomorphism class of loops with different scale; however, there is only one term surviving in $\sum_{a(v)\in[a(v)]}<\hat{H}^{\epsilon'}_v\Pi_s|f_\beta>_{Kin}$ since the graph β is fixed.

So, $\hat{\mathrm{M}}$ is a *positive* and *symmetric* operator in $\mathcal{H}_{Diff}$.

Note that the result of $< \eta(f_\beta)|\hat{\mathbf{M}}|\eta(g_\alpha) >_{Diff}$ coincides with the quadratic form $Q_{\mathrm{M}}(\eta(f_\beta), \eta(g_\alpha))$ defined by Thiemann [gr-qc/0305080] on (a dense form domain of) $\mathcal{H}_{Diff}$.

Hence, being the quadratic form associated with $\hat{\mathrm{M}}$, Q_{M} is closable. The closure of Q_{M} is the quadratic form of a unique self-adjoint operator $\overline{\hat{\mathrm{M}}}$, called the Friedrichs extension of $\hat{\mathrm{M}}$.

We relabel $\overline{\hat{\mathrm{M}}}$ to be $\hat{\mathrm{M}}$ for simplicity.

In conclusion, there exists a positive and self-adjoint operator $\hat{\mathrm{M}}$ on $\mathcal{H}_{Diff}$ corresponding to the master constraint.

3. Discussion and Outlook

- Discussion

  ⋆ Can one use the direct integral decomposition (DID) of $\mathcal{H}_{Diff}$ associated with $\hat{\mathbf{M}}$ to obtain $\mathcal{H}_{phys}$?

  Yes, since $\hat{\mathbf{M}}$ is self-adjoint, and there is a separable subspace of $\mathcal{H}_{Diff}$ which is left invariant by $\hat{\mathbf{M}}$ and captures the full physics of LQG [Thiemann, gr-qc/0510011]. Otherwise one may consider a separable $\mathcal{H}_{Diff}$ introduced by suitable extension of diffeomorphism transformations [Fairbairn and Rovelli, JMP 45(2004), 2802].

  ⋆ Can one identify $\mathcal{H}_{phys} = \mathcal{H}^{\oplus}_{\lambda=0}$ with the induced physical inner product $< \mid >_{\mathcal{H}^{\oplus}_{\lambda=0}}$?

  Yes, since zero is in the spectrum of $\hat{\mathbf{M}}$ [Thiemann, gr-qc/0510011].

  ⋆ How about the issue of quantum anomaly?

  It is expected to be represented in terms of the size of $\mathcal{H}_{phys}$ and the existence of sufficient semi-classical states.

  ⋆ Has the master constraint program been well tested?

  Yes, in various examples [Dittrich and Thiemann: gr-qc/0411138, gr-qc/0411139, gr-qc/0411140, gr-qc/0411141].

⋆ Trouble and the way out:

The expression of $\hat{\mathbf{M}}$ is so complicated that it is difficult to obtain the DID representation of $\mathcal{H}_{Diff}$ directly.

Fortunately, the subalgebra generated by master constraints is an Abelian Lie algebra in the master constraint algebra. So one can employ group averaging strategy to solve the master constraint.

Since $\hat{\mathbf{M}}$ is self-adjoint, by Stone's theorem there exists a strong continuous one-parameter unitary group,

$$\hat{U}(t) := \exp[it\hat{\mathbf{M}}],$$

on $\mathcal{H}_{Diff}$. Then, given any diffeomorphism invariant cylindrical functions $\Psi_{Diff} \in Cyl^{\star}_{Diff}$, one can obtain algebraic distributions of $\mathcal{H}_{Diff}$ by a rigging map η_{phys} from $Cyl^{\star}_{Diff}$ to Cyl_{phys},

$$\eta_{phys}(\Psi_{Diff})[\Phi_{Diff}] := \int_{\mathbf{R}} \frac{dt}{2\pi} < \hat{U}(t)\Psi_{Diff}|\Phi_{Diff} >_{Diff},$$

which are invariant under the action of $\hat{U}(t)$ and constitute a subset of the algebraic dual of $Cyl^{\star}_{Diff}$.

- Ongoing work

  ⋆ Calculate the physical inner product

  It is defined formally as

$$
\begin{aligned}
&< \eta_{phys}(\Psi_{Diff})|\eta_{phys}(\Phi_{Diff}) >_{phys} := \eta_{phys}(\Psi_{Diff})[\Phi_{Diff}] \\
&= \int_{\mathbf{R}} \frac{dt}{2\pi} < \hat{U}(t)\Psi_{Diff}|\Phi_{Diff} >_{Diff} .
\end{aligned}
$$

  Calculate the integrand

$$
\begin{aligned}
& < \hat{U}(t)\Psi_{Diff}|\Phi_{Diff} >_{Diff} \\
= & < \Psi_{Diff}|\exp(-it\hat{\mathbf{M}})|\Phi_{Diff} >_{Diff} \\
= & \lim_{N\to\infty} < \Psi_{Diff}|[\exp(-it\frac{\hat{\mathbf{M}}}{N})]^N|\Phi_{Diff} >_{Diff} \\
= & \lim_{N\to\infty} \sum_{[s_1]\ldots[s_{N-1}]} < \Psi_{Diff}|\exp[-it\frac{\hat{\mathbf{M}}}{N}]|\Pi_{[s_1]} >_{Diff} \times \\
& < \Pi_{[s_1]}|\exp[-it\frac{\hat{\mathbf{M}}}{N}]|\Pi_{[s_2]} >_{Diff} \times \\
& \ldots < \Pi_{[s_{N-2}]}|\exp[-it\frac{\hat{\mathbf{M}}}{N}]|\Pi_{[s_{N-1}]} >_{Diff} \times \\
& < \Pi_{[s_{N-1}]}|\exp[-it\frac{\hat{\mathbf{M}}}{N}]|\Phi_{Diff} >_{Diff} .
\end{aligned}
$$

One may consider the strategy of a possible approximate calculation:

$$
\begin{aligned}
& < \Pi_{[s]}|\exp[-it\frac{\hat{\mathbf{M}}}{N}]|\Pi_{[s']} >_{Diff} \\
= & < \Pi_{[s]}|1 - it\frac{\hat{\mathbf{M}}}{N}|\Pi_{[s']} >_{Diff} + O(\frac{1}{N^2}) \\
= & \ \delta_{[s][s']} - \frac{it}{N} < \Pi_{[s]}|\hat{\mathbf{M}}|\Pi_{[s']} >_{Diff} + O(\frac{1}{N^2}) \\
= & \ \delta_{[s][s']} - \frac{it}{N} Q_{\mathbf{M}}(\Pi_{[s]}, \Pi_{[s']}) + O(\frac{1}{N^2}).
\end{aligned}
$$

$\star$ Semiclassical analysis

Since the Hilbert spaces $\mathcal{H}_{Kin}$, $\mathcal{H}_{Diff}$, and the operator $\hat{\mathbf{M}}$ are constructed in such ways that are drastically different from usual quantum field theory, one has to check whether the constraint operators and the corresponding algebra have correct classical limits with respect to suitable semiclassical states.

To do the semiclassical analysis, we still need diffeomorphism invariant semiclassical states in $\mathcal{H}_{Diff}$. The research in this aspect is now in progress (There are positive results in simple models [Thiemann el]).

Quantum Gravity at BNU

- Gravity Group in Beijing Normal Univ, Beijing, CHINA
  - ⋆ The biggest theoretical relativity group in China: 8 professors (3 retired), around 20 graduate or doctoral students.
  - ⋆ Research Area: Black hole thermodynamics, Classical GR, Cosmology, High dimensional gravity, Loop quantum gravity.
- LQG in Beijing Normal Univ
  - ⋆ Professors: Weiming Huang (Algebraic geometry, Quantum gravity), Yongge Ma (LQG, High dimensional gravity), Thomas Thiemann (Visiting professor).
  - ⋆ Graduate students: You Ding, Li Qin, Li-e Qiang, Peng Xu, Jinsong Yang, Hua Zhang.
  - ⋆ Review article: M. Han, W. Huang, and Y. Ma, Fundamental structure of loop quantum gravity, gr-qc/0509064. (Welcome comments and suggestions!)
- Welcome your communication and cooperation!

Thank you!

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