# A Discrete Machian Model

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*The motion of a body K can only be estimated by reference to other bodies A, B, C, ... When we reflect that we cannot abolish the isolated bodies A, B, C, ... that is, cannot determine by experiment whether the part they play is fundamental or collateral, that hitherto they have been the sole and only competent means of the orientation of motions ..., it will be found expedient provisionally to regard all motions as determined by these bodies.*

- E. Mach *Science of Mechanics*

# A Discrete Machian Model

Outline:

- Motivation:
  - LQG Discrete Geometric Spectra
  - Two hypotheses
  - Cellular froth
  - Machian effects
- Discrete Machian Model
  - Lagrangian and non-rel Hamiltonian
  - Example test: Constraining Machian parameters Spin-Polarized torsion pendulum
- Summary

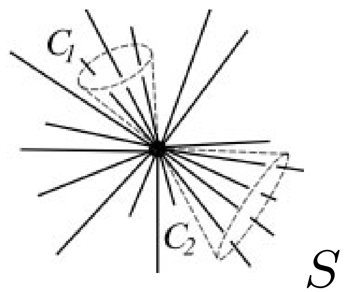
# LQG: Discrete Spatial Geometry

**Area**: $\widehat{A}_S \mid s\rangle = a \mid s\rangle$

$$a = \ell_P^2 \sum_{n=1}^{N} \sqrt{j_n(j_n+1)}$$

**Angle**: $\widehat{\theta} \mid s\rangle = \theta \mid s\rangle$

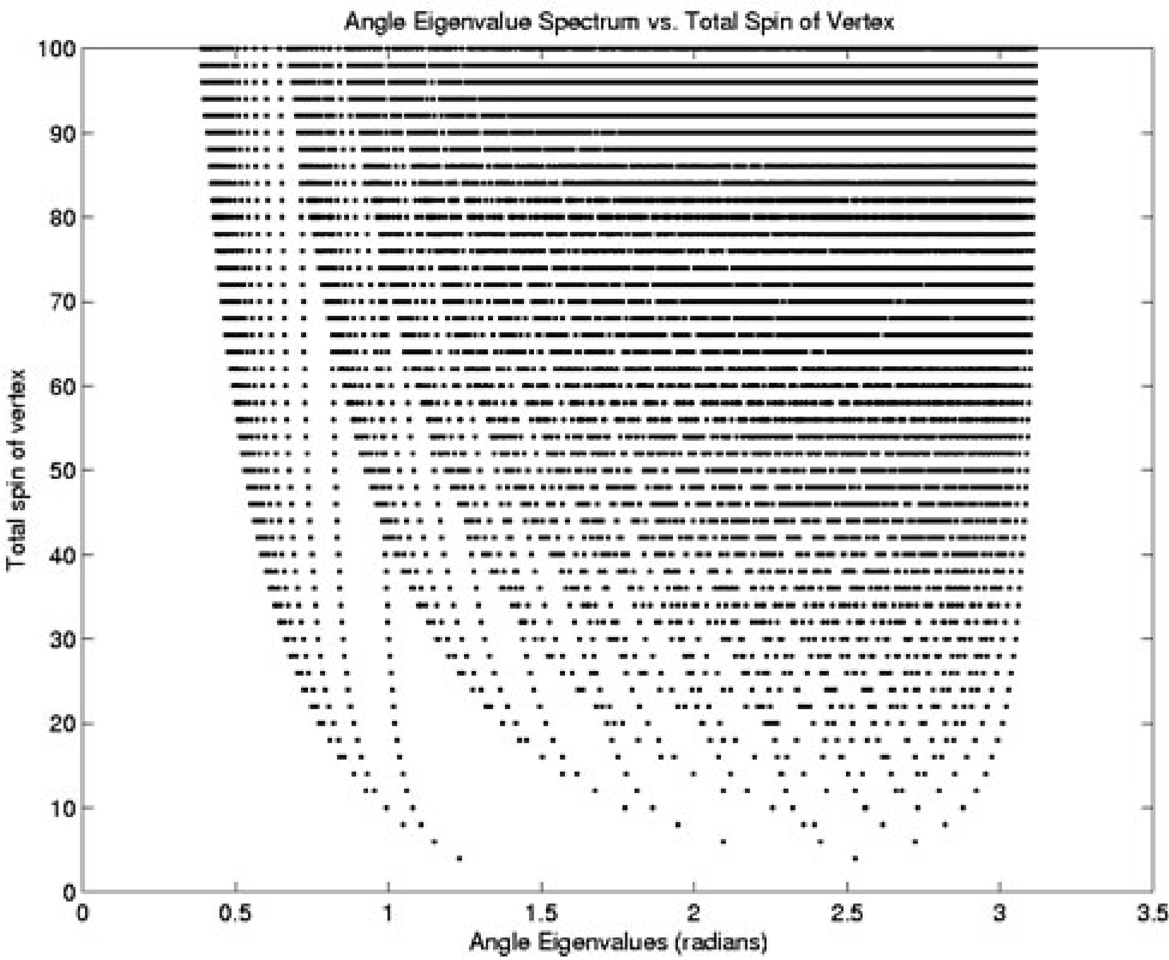
$$\theta = \text{arccos}\left(\frac{j_r(j_r+1) - j_1(j_1+1) - j_2(j_2+1)}{2\,[j_1(j_1+1)\,j_2(j_2+1)]^{1/2}}\right)$$




Rovelli,Smolin NPB **422** (1995) 593; Asktekar,Lewandowski CQG **14** (1997) A43

SM CQG **16** (1999) 3859

# Discrete spectra for Geometry




One vertex studies "continuum angles approximated" for total vertex spins of

$$j \sim 10^{20} \,!$$

[SM, Siefert CQG **19** (2002) 2211]

# Discrete Geometry Phenomenology

$\rightsquigarrow$ Deep geometry has no metric structure

## Discrete Geometry Phenomenology

$\rightsquigarrow$ Deep geometry has no metric structure

- Metric is emergent
- Local Lorentz invariance is "broken" (LLI)
- Local position invariance is "broken" (LPI)
- Rotational invariance is "broken"
- Remnants of deep spatial geometry have physical effects
- Effects exist in flat space, curvature is not necessary

# Discrete Geometry Phenomenology

$\rightsquigarrow$ Observed geometry is stable

$\rightsquigarrow$

# Discrete Geometry Phenomenology

$\rightsquigarrow$ Observed geometry is stable

- no inflating extra dimensions
- no sign of signature change
- no change in orientation
- Planck temperature $10^{32}$ K
- Fluctuations small, long range

$\rightsquigarrow$ Deep geometry stable and discrete

# Discrete Geometry Phenomenology

In LQG one would like semiclassical states of flat space.

Simple model of deep spatial geometry "Cellular froth":
- Random geometry is "close" to LPI, LLI, rotational invariance
- no regular complex
- grainularity: average coordination number or "contiguity number"
- For Voronoi (or Wigner-Seitz) froth average faces/ cell 13.6 (Coexeter)

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$\rightsquigarrow$ fundamental particle motion is stochastic

# A Machian Model

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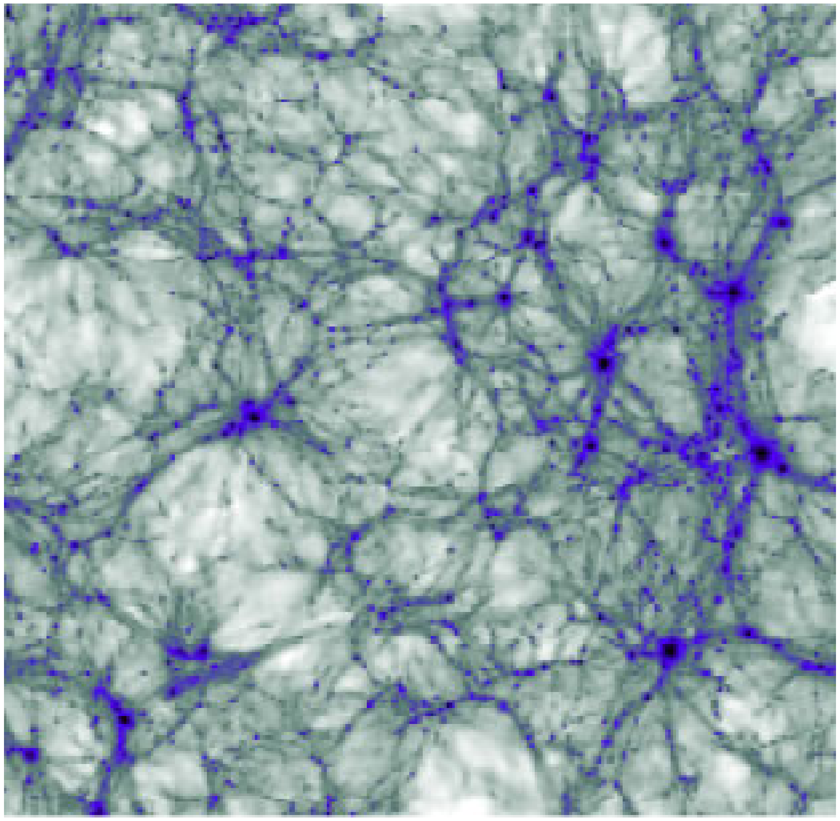
Non-uniformities lead to a **preferred direction**



Springel, Hernquist, White (2000)

Cocconi-Salpeter's "Mach's principle"

$m \to m_{ij}$ e.g. $F^i = m_{ij} a^j$

where $m_{ij} = m\delta_{ij} + \Delta m_{ij}$

$$\Delta m \propto \frac{M(r)}{r^{\nu}}$$
$$\Delta m \propto f(\theta)$$

# Discrete Machian Model

$\rightsquigarrow$ There is a local, dynamic preferred direction.

- Remnant of discrete Machian space, a direction field $u^\mu$
- spatial (vs. Jacobson and Mattingly)
- local preferred direction
- determined by anisotropies in matter distribution
- stochastic
- physical effects scale with distance to source of anisotropy
- spatial $< u^\mu >$ retains Machian effects, looses discrete effects

# Discrete Machian Model: Fermions

$\rightsquigarrow$ Continuum/metric approximation is very accurate

- can use Effective Field Theory
- remnant effects of discrete geometry are tiny - use PT

Goal is to constrain parameters and explore tests:

- LV leads to the possibility of strong constraints, e.g. bounds on modifications to dispersion relations
- Violation of rotational invariance leads to low energy effects

# Discrete Machian Model: Fermions

Effective Field Theory model:

- flat metric $\eta_{\mu\nu}$
- inertial mass parameter $m$ determined by the overall matter distribution

All possible dimension 4 operators linear in the modifications, constructed from $u^{\mu}$

- linear in DMM parameters
- $u$ dynamics not included in the field theory

# Discrete Machian Model: Fermions

Effective Field Theory model

Relativistic lagrangian, free spin-1/2 fermion $\psi$

$$\mathcal{L} = i\bar{\psi}\Gamma^{\mu}\partial_{\mu}\psi + \bar{\psi}M\psi$$

with

$$\Gamma^{\mu} = \gamma^{\mu} + \alpha\theta^{\mu\nu}\gamma_{\nu} + \beta\theta^{\mu\nu}\gamma_5\gamma_{\nu} + \delta u^{\mu} + i\epsilon\gamma_5 u^{\mu}$$

and

$$M = m + \zeta u^{\mu}\gamma_{\mu} + \eta\gamma_5\gamma_{\mu}u^{\mu}$$

- $\theta^{\mu\nu} = \frac{1}{2}u^{(\mu}u^{\nu)}$
- $u$ stochastic
- "universal" effect

## Discrete Machian Model: Fermions

Low energy, high precision LLI tests - non-relativistic theory
- Foldy-Wouthuysen transformation expansion in $p/m$ (modified by Kostelecky and Lane hep-th/9909542)

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Low energy, high precision LLI tests - non-relativistic theory

- Foldy-Wouthuysen transformation expansion in $p/m$ (modified by Kostelecky and Lane hep-th/9909542)

To order $p/m$ and linear in DMM parameters:

$$
\begin{aligned}
H_{NR} &= \frac{1}{2m}\left(\delta^{ij} + \delta m^{ij} + \delta m_l^{ij}\sigma^l\right)(p_i + eA_i)(p_j + eA_j) \\
&+ \left(\delta n^i + \delta_l^i \sigma^l + \beta\theta_{00}\sigma^i\right)(p_i + eA_i) \\
&+ \frac{e}{2m}(1 - 2\alpha\theta_{00})\sigma^i B_i + \frac{e\alpha}{2m}\epsilon^{ijk}\sigma_i\theta_{lk}\partial_j A_l + \delta m + \delta m^i \sigma_i - e\phi
\end{aligned}
$$

Effects:

- Anisotropic inertial mass
- Spin-coupled mass

# Discrete Machian Model: Fermions

$$
\begin{aligned}
\delta m_{ij} &= -2\alpha\theta_{00}\delta_{ij} + 2\alpha\theta_{ij} \\
\delta m_{ij}^{l} &= 2\beta\theta_{j0}\delta_i^l + i\alpha\epsilon_j^{lk}\theta_{ik} \\
\delta n_i &= -\alpha\theta_{j0} - \delta u_j + i\alpha\frac{\partial_j\theta_i^j}{2m} - \frac{\zeta u_i}{m} \\
\delta n_i^l &= \beta\theta_i^l + \alpha\frac{\epsilon^{ljk}\partial_j\theta_{ik}}{2m} - i\beta\frac{\partial^l\theta_{i0}}{2m} - \epsilon\frac{\partial^l u_i}{2m} - \eta\frac{u_0\delta_i^l}{m} \\
\delta m &= \zeta u_0 + \delta u_0 m - \alpha m\theta_{00} + i\zeta\frac{\partial_j u^j}{2m} \\
\delta m^l &= -\eta u^l - \beta m\theta_0^l + \zeta\frac{\epsilon^{ljk}\partial_j u_k}{2m} + i\eta\frac{\partial^l u_0}{2m}
\end{aligned}
$$

- frame with $u^0 = 0$ effects still present.

# Discrete Machian Model: Tests

Example: Spin-polarized torsion pendulum

Hamiltonian contains term $\delta m^l \sigma_l$

Experiment is macroscopic, $S = 8 \times 10^{22}$ (U. Washington)
- spatially and temporally averaged data
- turntable rotated at $\omega$
- look for signals at $\omega$, $\Omega \pm \omega$ , $\Omega$ sidereal frequency
- bounds on $\delta m_z \sim 10^{-28}$ GeV (coherent)
- In a minimal model where the DMM parameters are all determined by the Cocconi-Salpeter scaling $M/r^\nu$ the sun gives a fall off for the Machian effects: $\nu \leq 0.15$

## Discrete Machian Model: Summary

$\rightsquigarrow$ New phenomenology of discrete geometry

- Remnant of discrete space via a stochastic direction field $u$
- EFT model for fermions
- universal effects
- parameters granularity, machian fall off with distance
- stochastically broken LLI

# Discrete Machian Model: Summary

More tests:
- Penning trap (single electron)
- Clock comparison experiments
- High energy tests
- non-systematic dispersion relations
- particle production

Differs from extended standard model (Kostelecky et. al.)
- "dynamical" breaking due to (stochastic) field
- fewer parameters
- universal effects