The thermal time hypothesis: overview and application

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C. Rovelli, P.M. Class. Quant. Grav **20** (2003) 4919-4932

P.M. gr-qc/0401116

Question of time in quantum gravity: the flow of physical time is a state dependent notion (Rovelli 1993).

"Von Neumann algebras naturally evolve with time" (Connes).

Unruh effect: uniformly *and eternal* accelerated observer sees the vacuum of a quantum field theory on Minkovski spacetime as a thermal state.

Question: How to intrinsically characterize the flow of time in a generally covariant context ?

How to relate the freedom of choosing any variable as the time variable with the intuition of a prefered (at least locally) unique parameter measuring the local time flow ?

Possible answer: the flow of time is determined by the thermodynamical state of the system.
From lack of information (knowledge = statistical) emerges a flow which coincides to what we perceive as time.

C. Rovelli Class. Quant. Grav. **10** (1993) 1549-1566.

Definitions of time and thermal state are linked:

Let $\mathcal{A}$ be the algebra of observables of a system, α_t be the time evolution (e.g. $\alpha_t a = e^{-iHt} a e^{iHt}$) then an equilibrium state ω at temperature β^{-1} is a state that satisfies the KMS condition

$$\omega((\alpha_t a)b) = \omega(b(\alpha_{t+i\beta} a)).$$

Equivalent to Gibbs definition ($\omega(a) = \frac{1}{Z}\mathrm{Tr}(e^{-\beta H} a)$) and still make sense at the thermodynamical limit (whereas Z is no longer defined).

> An equilibrium state at temperature β^{-1} is a state that satisfies the KMS condition with respect to the time evolution α_t.

Von Neumann algebras naturally evolve with time:

$$\left.\begin{array}{l}\text{- a von Neumann algebra } \mathcal{A} \text{ acting on } \mathcal{H} \\ \\ \text{- a vector } \Omega \text{ in } \mathcal{H} \text{ cyclic and separator} \\ (\mathcal{A}\Omega \text{ and } \mathcal{A}'\Omega \text{ are dense in } \mathcal{H})\end{array}\right\} \Rightarrow \begin{array}{c}\text{a 1-parameter group } \sigma \\ \text{of automorphisms of } \mathcal{A} \\ \text{(modular group of} \\ \text{Tomita-Takesaki)}\end{array}$$

The state $\omega : a \mapsto \langle \Omega, a\Omega \rangle$ is KMS with respect to σ_s,

$$\omega((\sigma_s a)b) = \omega(b(\sigma_{s-i} a)) \quad \forall a, b \in \mathcal{A},\ s \in \mathbb{R}.$$

Hence ω is a thermal state with respect to the evolution σ_s.

> An equilibrium state at temperature β^{-1} is a faithful state over the algebra of observables whose modular group σ_s is the physical time translation, up to rescaling
>
> $$t = -\beta s.$$

$$\left\{\begin{array}{l}\text{time flow } \alpha_t \\ \text{temperature}\end{array}\right. \xRightarrow[\text{KMS}]{} \text{equilibrium state}$$

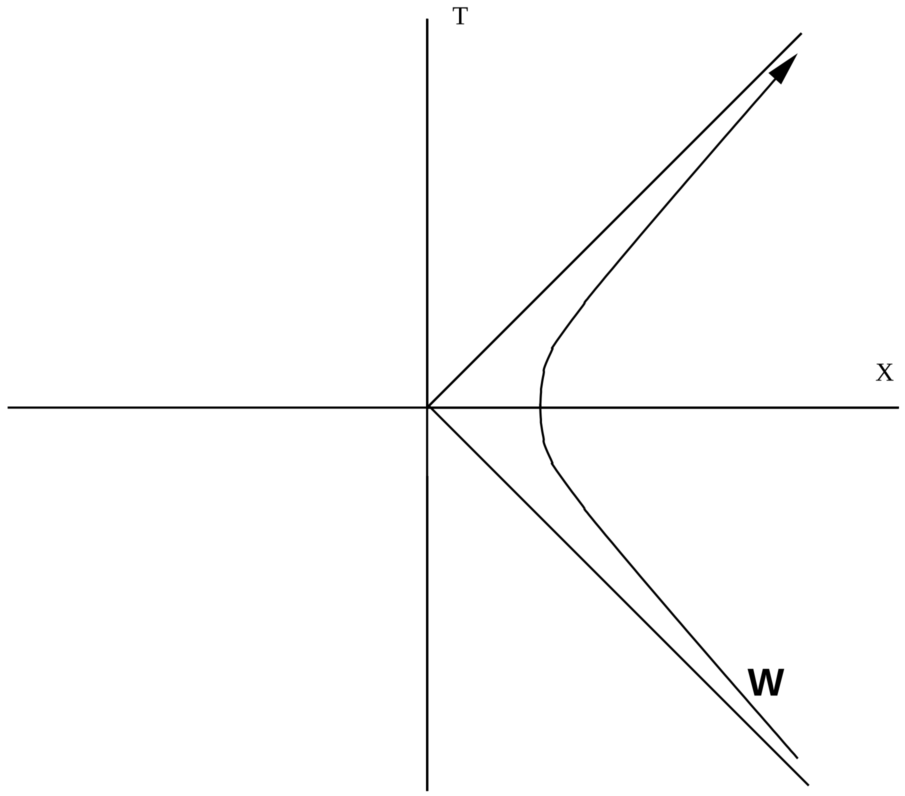
$$\left\{\begin{array}{l}\text{state } \omega \\ \text{temperature}\end{array}\right. \xRightarrow[\text{modular theory}]{} \text{time flow}$$

<u>The thermal time hypothesis</u>(Connes, Rovelli 1993): assuming the system is in a thermal state at temperature β^{-1}, then the physical time t is the modular flow up to rescaling $t = -\beta s$.

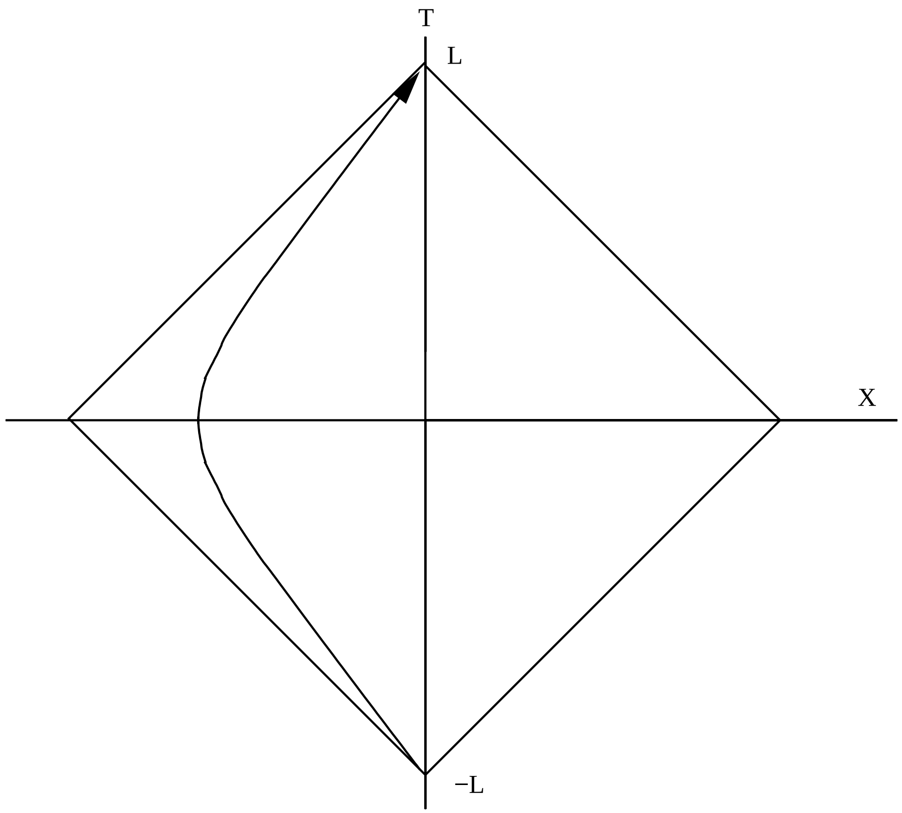
If another notion of time is available (e.g. a geometrical time τ), one should check that $\tau = t$, i.e. $\beta = \frac{-\tau}{s}$.

$$\left\{\begin{array}{l}\text{state} \\ \text{time}\end{array}\right. \Longrightarrow \text{temperature}$$

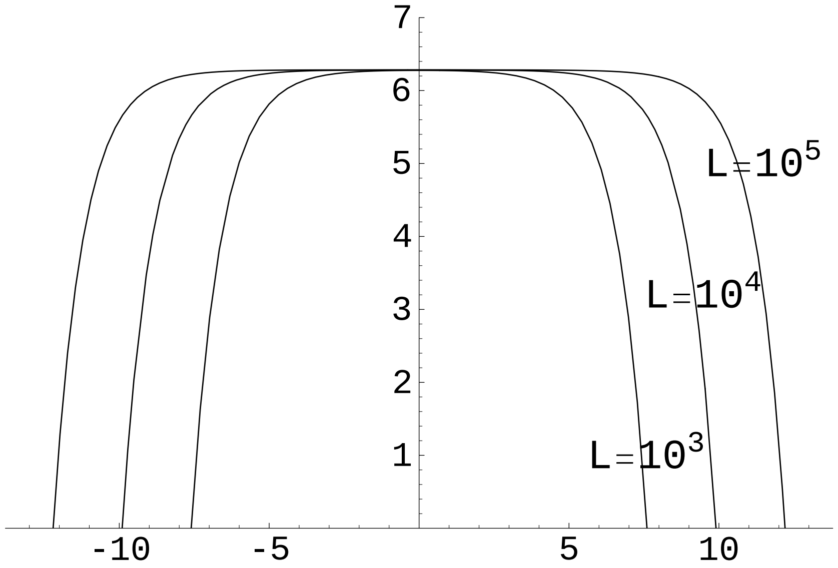
Unruh effect: uniformly accelerated observer O lies within W. Axiomatic qft: $W \mapsto \mathcal{A}(W)$. Vacuum is KMS with respect to O's proper time. Temperature $T_U = \frac{\hbar a}{2\pi k_b c}$ (Bisognano, Wichman).



Non eternal observer: $\beta(L,\tau) = -\frac{d\tau}{ds} = \frac{2\pi}{La^2}(\sqrt{1+a^2L^2} - \cosh a\tau)$.



Interpretation: $\beta(L,\tau)^{-1} \approx \beta(0,L)^{-1} = T_U(1 - \frac{1}{aL} + \mathcal{O}(\frac{1}{L^2}))$.



For O' whose clock would measure $t = -\beta(0,L)s$, the vacuum is thermal. For most of his lifetime, the proper time τ of O coincides with t. Hence the vacuum is thermal for O ?

Hence a natural way to associate a temperature

$$T_D(a, L) = \beta(0, L)^{-1}$$

to a diamond region D of Minkovski spacetime. Moreover T_D <u>does not vanish for an inertial observer</u>,

$$T_D(0, L) = \frac{1}{\pi L}.$$

Causal problem since T_D depends on the lifetime $\tau_0 = f(L)$. For the measurement to be accurate enough, one needs enough time:

$$\frac{\hbar}{k_b T_D} \leq \Delta\tau \leq \tau_0 \text{ i.e. } T_D \geq \frac{\hbar}{k_B \tau_0}.$$

Not possible for small and large a. For intermediate a, one is saved by

$$|T_D - T_U| \geq \frac{\hbar}{k_B \tau_0}.$$

Conclusion: the correction to Unruh temperature is causally acceptable only if it is physically undetectable... Anyway T_U is already very small ($a = 10^9 ms^{-2}$ for $T_U = 1K$).

BUT

- origin of the temperature is geometrical (and not due to the acceleration).

- by conformal transformation, the asymptotic limit (i.e. the eternity of the observer) is mapped to a *sharp* divergence of the temperature. Could be interesting in Kerr spacetime (Killing field has spacelike orbits near infinity) for the identification to the Hawking temperature.

- adaptable to curved spacetime, e.g. de Sitter (Y. Tian 2005).

- in all known examples, the thermal time hypothesis makes sense. Good sign for its interest for quantum gravity ? (which algebra of observables ? Which state ?).