Parametrised Group Field Theories and Quantum Gravity transition amplitudes

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Plan of the talk[a]

- **why**?
  1. why new models?
  2. why orientation?
  3. why a GFT derivation?
- orientation-dependent/causal spin foam models
- (briefly) GFT for the Barrett-Crane model
- **new generalised/parametrised GFTs**
- **what next**?

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[a] Focus here is on 4d BC model, but results are general

What is the aim?

Construct a generalised formalism

for Group Field Theories

from which one can derive
orientation-dependent/causal spin foam models

(and recover usual models as well)

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- understand/make link with other approaches (causal dynamical triangulations, causal sets, loop quantum gravity, etc)
- different transition amplitudes for quantum gravity (different spin foam models may be different amplitudes/quantities in the same theory (as in QFT)

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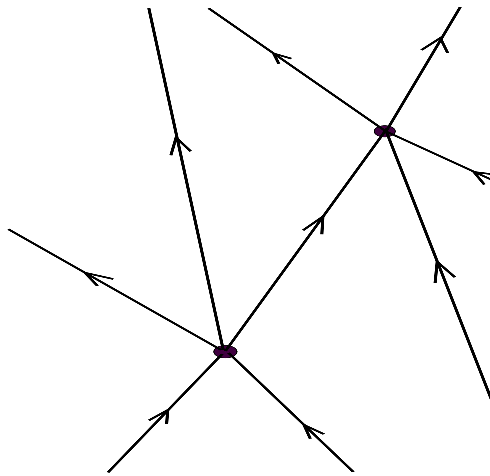

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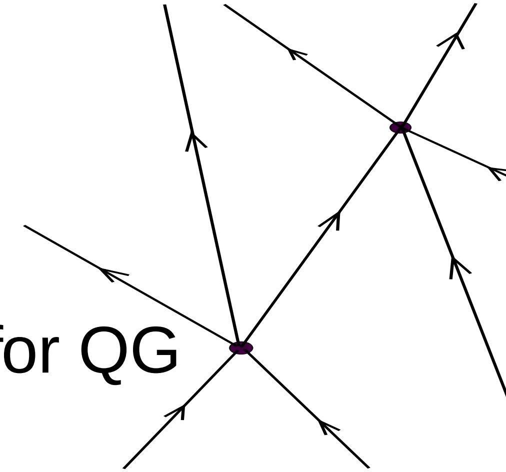

- causal/a-causal transition amplitudes for QG

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GFT provides a sum over 2-complexes/triangulations (needed for full sum over histories of spin networks, gets rid of triangulation dependence

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- Quantum Gravity as an (almost) ordinary QFT (with a background spacetime given by the group manifold)
- door towards non-perturbative properties of QG using ordinary QFT techniques

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- Causal Sets: GFT sums over directed graphs and should provide orientation-dependent amplitudes

Orientation-dependent/causal spin foam models

Barrett-Crane model (group $G = Spin(4)$ or $G = SL(2,\mathbb{C})$, 2-complex Γ):

$$Z = \sum_{\Gamma} \lambda(\Gamma) \sum_{\{J_f\}} \prod_f \Delta_{J_f} \prod_e A_e(\{J_{f(e)}\}) \prod_v A_v^{BC}(\{J_{f(v)}\})$$

J = unitary irreps of G, $\Delta_J =$dimension irrep$J$, f$\leftrightarrow$triangles, e$\leftrightarrow$tetrahedra, v$\leftrightarrow$4-simplices, $\Gamma \leftrightarrow$ triangulation

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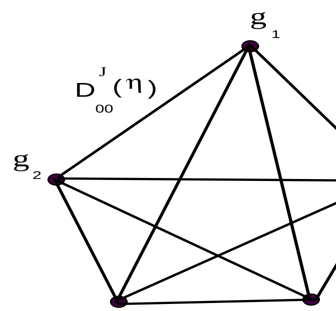

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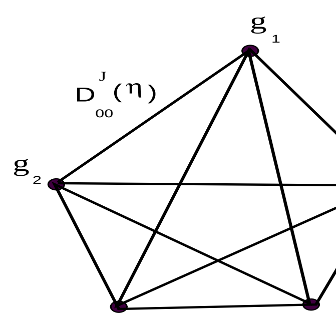

$$D_{00}^J(\eta) = \frac{\sin\sqrt{\Delta_J}\eta}{\sqrt{\Delta_J}\sin(h)\eta} = \frac{e^{i\sqrt{\Delta_J}\eta}}{2i\sqrt{\Delta_J}\sin(h)\eta} - \frac{e^{-i\sqrt{\Delta_J}\eta}}{2i\sqrt{\Delta_J}\sin(h)\eta}$$

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- Barrett-Crane model does not register orientation of 2-complex/triangulation, given by an assignment of $\alpha_{e(v)} = \pm 1$ (with $\alpha_{e(v_1)} = -\alpha_{e(v_2)}$) to edges and $\epsilon_{f(v)} = \alpha_{e_1}\alpha_{e_2} = \pm 1$ to faces

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- Due to sum of two exponentials in each $D^J_{00}(\eta)$, that makes amplitudes real, while amplitudes for opposite orientations should be related by complex conjugation (dual representations); each corresponds to one possible orientation ϵ_f of face; this is origin of cosine of Regge action in asymptotics of 4-simplex amplitude

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- Construct oriented models restricting consistently amplitudes to include just one exponential; should give directly exponential of Regge action; Livine-Oriti, 2002

Orientation-dependent/Causal spin foam models

Refined realisation based on particle analogy (Oriti, 2004):

- $D^J_{00}(gg'^{-1})$ is Hadamard propagator for particle on group manifold, a-causal sum of two (time) oriented Wightman functions (the two exponentials $E^J_{\pm}(\eta)$), with mass $m^2 = -C_J$

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- Construct oriented models by using instead Feynman propagators on G for defining the amplitudes
- Construction uses evolution kernel in proper time:

$$H(g,g',m^2) = \int_{\mathbb{R}} ds K(g,g';s)\, e^{im^2 s} \propto D^J_{00}(gg'^{-1}), \Delta_J = 1$$

$$G(g,g',m^2) = \int_{\mathbb{R}} ds\theta(\epsilon s) K(g,g';s)\, e^{im^2 s} \propto E^{m^2}_{\epsilon}(gg'^{-1})$$

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$$\int_{\mathbb{R}} ds K(g, g'; s)\, e^{im^2 s} = \int_{\mathbb{R}} ds \sum_J \Delta_J D^J_{00}(gg'^{-1}) e^{i(C_J + m^2)s}$$

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- analogously:

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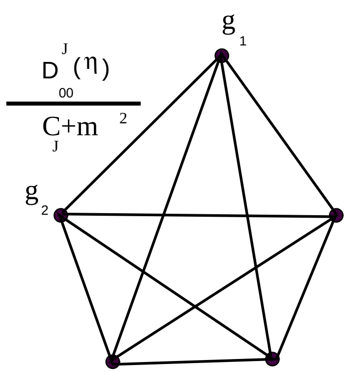
$$\rightarrow \frac{D^J_{00}(gg'^{-1})}{C_J + m^2 + i\epsilon\delta} \simeq \frac{D^J_{00}(gg'^{-1})}{\sqrt{\Delta_J} + \sqrt{1 - m^2}} + \frac{D^J_{00}(gg'^{-1})}{\sqrt{\Delta_J} - \sqrt{1 - m^2}}$$

Orientation-dependent/Causal spin foam models

Amplitudes we want to get, for given 2-complex, in full momentum space (2 variables (J, m^2) for each face):

$$Z(\Gamma) = \left(\prod_f \sum_{J_f} \int_{\mathbb{R}} dm_f^2\right) \prod_f A_f(J_f, m_f^2) \prod_e A_e(J_{f(e)}, m^2_{f(e)})$$

$$\prod_v \left(\prod_e \int_G dg_{e(v)} \prod_{f(v)} \frac{D^{J_f}_{00}(g_{e1} g_{e2}^{-1})}{C_{J_f} + m_f^2}\right)$$



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define also projector $P_h : P_h \phi(g_1, g_2, g_3, g_4) =$
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Define classical theory by action:

$$S[\phi] = \frac{1}{2} \int dg_1..dg_4 [P_g \left(P_h\right) \phi(g_1, g_2, g_3)]^2 \; -$$

$$-\frac{\lambda}{5!} \int dg_1..dg_{10} [P_g P_h \phi(g_1, g_2, g_3, g_4)][P_g P_h \phi(g_4, g_5, g_6, g$$

$$[P_g P_h \phi(g_7, g_8, g_2, g_9)] P_g P_h \phi(g_9, g_3, g_5, g_{10})][P_g P_h \phi(g_{10}, g_6, g_8, g$$

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- with A_e^1 (DP-F-K-R version) or A_e^2 (P-R version)

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1) group elements $\leftrightarrow$ irreps of G, proper time parameter $\leftrightarrow$ mass variable
$\longrightarrow$ field theory over group manifold with extra proper time independent coordinate and a variable mass (conjugate to proper time) (Fock, Feynman, Nambu, Stueckelberg,...)
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momentum space: $\phi(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4) =$

$= \sum_{J_i} \int_{\mathbb{R}} dm_1^2..dm_4^2\, \phi^{J_1 J_2 J_3 J_4}_{k_1 l_1 ... k_4 l_4}(m_1^2, .., m_4^2)$

$D^{J_1}_{k_1 l_1}(g_1)..D^{J_4}_{k_4 l_4}(g_4) e^{i m_1^2 s_1}..e^{i m_4^2 s_4}$

New parametrised GFT

usual symmetry:

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and define operator P_s:
$P_s \phi(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4) =$
$\int_{\mathbb{R}} ds\, \theta(s)\, \phi(g_1, s_1 + s; g_2, s_2 + s; g_3, s_3 + s; g_4, s_4 + s)$

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Denote $\phi^\alpha(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4)$ such that

$$\phi^+(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4) = \phi(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4)$$

and

$$\phi^-(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4) = \phi^\dagger(g_1, s_1; g_2, s_2; g_3, s_3; g_4, s_4)$$

New parametrised GFT

Consider the action ($\phi_{gs} = P_g P_h P_s \phi$):

$$S[\phi] = \prod_i \int_G dg_i \int_{\mathbb{R}} ds_i$$

$$\{P_g(P_h)P_s\phi^-(g_i,s_i)\left(\prod_i (i\partial_{s_i} + \nabla_i)\right) P_g(P_h)P_s\phi^+(g_i,s_i)+$$

$$+ P_g(P_h)P_s\phi^+(g_i,s_i)\left(\prod_i (-i\partial_{s_i} + \nabla_i)\right) P_g(P_h)P_s\phi^-(g_i,s_i)\}$$

$$+ \sum_{\{\alpha_i\}=\pm} \lambda_{\{\alpha_i\}} \prod \int_G dg_i \int_{\mathbb{R}} ds_i \left\{\phi_{gs}^{\alpha_1}(g_i^1,s_i^1)\phi_{gs}^{\alpha_2}(g_i^2,s_i^2)\phi_{gs}^{\alpha_3}(g_i^3,\right.$$

$$\phi_{gs}^{\alpha_4}(g_i^4,s_i^4)\phi_{gs}^{\alpha_5}(g_i^5,s_i^5)\prod_{i<j}\theta(\alpha_{e_i}\alpha_{e_j}(s_{ij}-\tilde{s}_{ij}))\,K(g_{ij},\tilde{g}_{ij};s_{ij}-$$

$$\lambda_{+++++} = \lambda^*_{-----}, \lambda_{++++-} = \lambda^*_{----+}, \lambda_{+++--} = \lambda^*_{---++}$$

Full amplitude for given 2-complex:

$$Z(\Gamma) = \left(\prod_f \sum_{J_f} \int_{\mathbb{R}} dm_f^2 \right) \prod_f \Delta_{J_f} \prod_e \tilde{A}_e(J_{f(e)}, m^2_{f(e)})$$

$$\prod_v \left(\prod_e \int_G dg_{e(v)} \prod_{f(v)} \frac{i\epsilon_f D^{J_f}_{00}(g_{e1} g_{e2}^{-1})}{C_{J_f} + m_f^2 + i\epsilon_f \delta} \right)$$

with

$$\tilde{A}^1_e = \left(\prod_{f(e)} \frac{1}{C_{J_f} + m_f^2} \right) A^1_e \qquad \tilde{A}^2_e = \left(\prod_{f(e)} \frac{1}{C_{J_f} + m_f^2} \right) f(m^2_{f_i}) A^2_e$$

New parametrised GFT

These parametrised GFTs generalise usual ones because of presence of extra variables s_i (m_i^2) $\rightarrow$ reduce to usual ones if no dependence on them

less trivial:
DP-F-K-R version of BC model recovered if: 1) drop dependence on orientation data in vertex term (no Theta functions); 2) go to *ultra-static* case:
$(i\partial_s + \nabla)\delta(g, g')\delta(s, s') \rightarrow \delta(g, g')\delta(s, s')$

P-R version is (almost) recovered in the same way

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- apply to 3d (Ponzano-Regge) case and study what changes of known results

What now?

If really exponential of Regge action comes out directly as amplitude, or in some limit, then really GFT can be seen as a general framework for most approaches to Non-Perturbative Quantum Gravity

$\Downarrow$

Loop Quantum Gravity, Causal Dynamical Triangulations, Causal Sets, Quantum Regge calculus

$\Downarrow$

It's time to study how it reduces to each of them, their relationships and differences, extablish solid links, construct bridges, understand role and usefulness of each