Isotropic universe dynamics from self-adjoint form of Hamiltonian constraint in LQC

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(work by Abhay Ashtekar, Parampreet Singh, TP)

Purpose of the talk

We construct and investigate numerically the semiclassical state corresponding to homogeneous and isotropic universe with massless scalar field quantized within Loop Quantum Cosmology framework. The following problems will be addressed:

- Flat universe:
  - Domain of validity of classical description.
  - Dynamics near classical singularity.
  - Behavior (classicality) 'on the opposite side' of the singularity.
- Closed (spherical) model:
  - The existence of semiclassical states corresponding to closed universe.

Properties of the model

- Isotropic universe with massless scalar field is in LQC described by Hamiltonian constraint $H_{\rm grav} + H_{\rm matter}$ where:
  - $\widehat{H}_{\rm grav}$: self-adjoint gravitational Hamiltonian

$$\widehat{H}_{\rm grav}\,|\mu\rangle = H^{+}(\mu)\,|\mu + 4\mu_0\rangle + H^{0}(\mu)\,|\mu\rangle + H^{-}(\mu)\,|\mu - 4\mu_0\rangle$$

  - $$\widehat{H}_{\rm matter} = \frac{\widehat{\Pi}^2}{2}\left(\widehat{\frac{sgn(p)}{\sqrt{|p|}}}\right)^3 := \frac{\widehat{\Pi}^2}{2}\widehat{B}.$$

    (Π - scalar field conjugate momentum)
  - Physical states represented by wave function $\psi(\mu,\phi)$ satisfying

$$(\widehat{B})^{-1}\widehat{H}_{grav}\,\psi(\mu,\phi) =: -\Theta\,\psi(\mu,\phi) = \frac{\partial^2}{\partial\phi^2}\psi(\mu,\phi)$$

  - ϕ is an internal clock

Calculation methods

- Only positive frequency solutions considered.
- Two possibilities of ψ calculation:
  - Specification of the initial data on some ϕ_0 and evolution along ϕ (ϕ-IVP),
  - specification of ψ at $\{\mu_{\rm in}, \mu_{\rm in} + 4\mu_0\}$ and 'evolution' by discrete steps in μ (μ-IVP).

  Second method technically easier.
- Due to observed parity invariance we are interested in solutions symmetric vs reflection in μ.
- For measurements we use the following Dirac observables:
  - $\widehat{\Pi}$ and
  - $\widehat{\mu_\phi}$ or $\widehat{\phi_\mu}$.

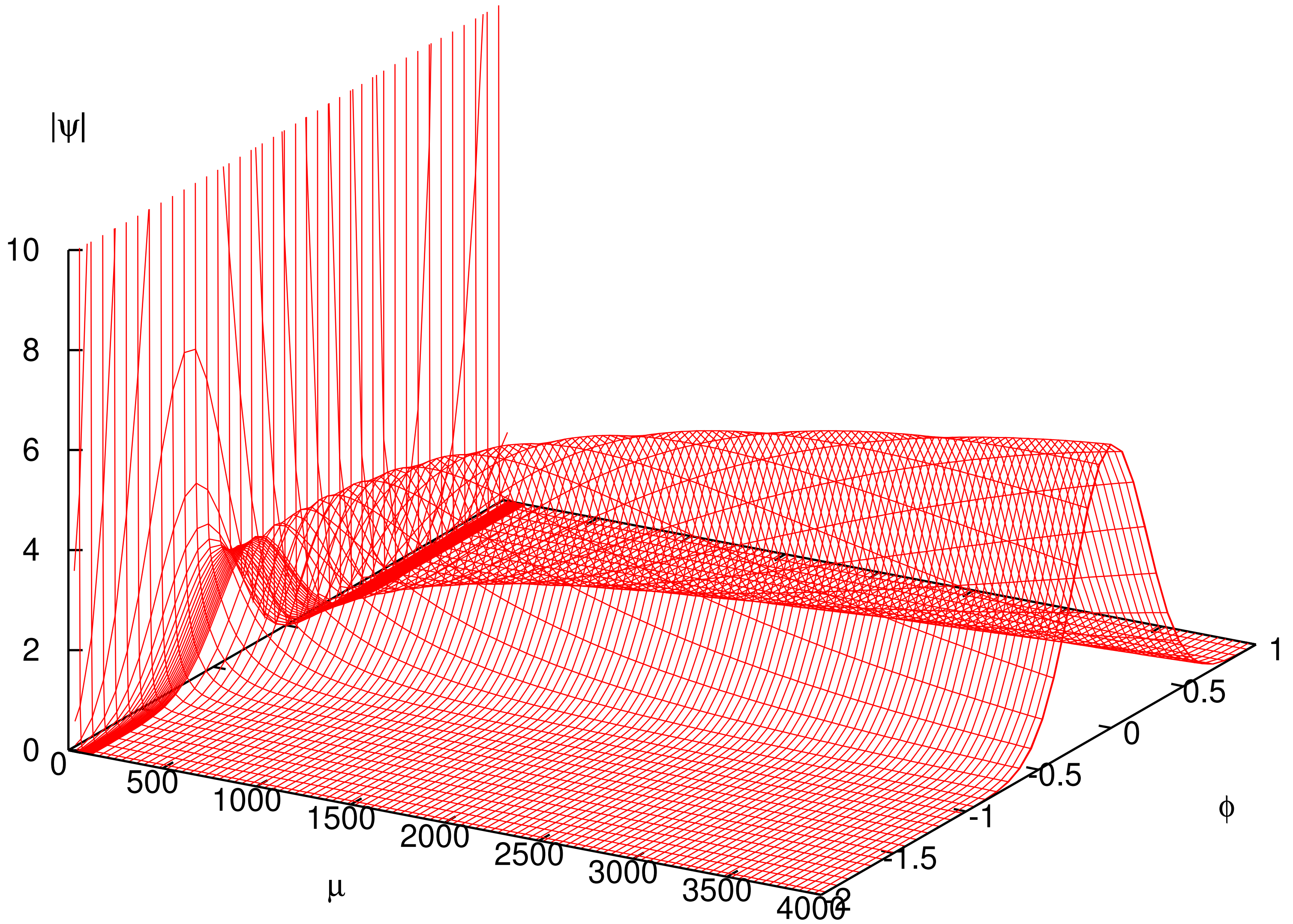
μ-IVP

- ψ calculated in Π representation $\psi = \psi(\mu, \Pi)$. Evolution of ψ reduces then to the evolution of the eigenfunctions of Θ ('plain waves').
- Initial conditions $(\psi(\mu_{\rm in}, \Pi), \psi(\mu_{\rm in} + 4\mu_0, \Pi))$ are specified to agree with Wheeler deWitt semiclassical state corresponding to an expanding spacetime (of the momentum Π_0)

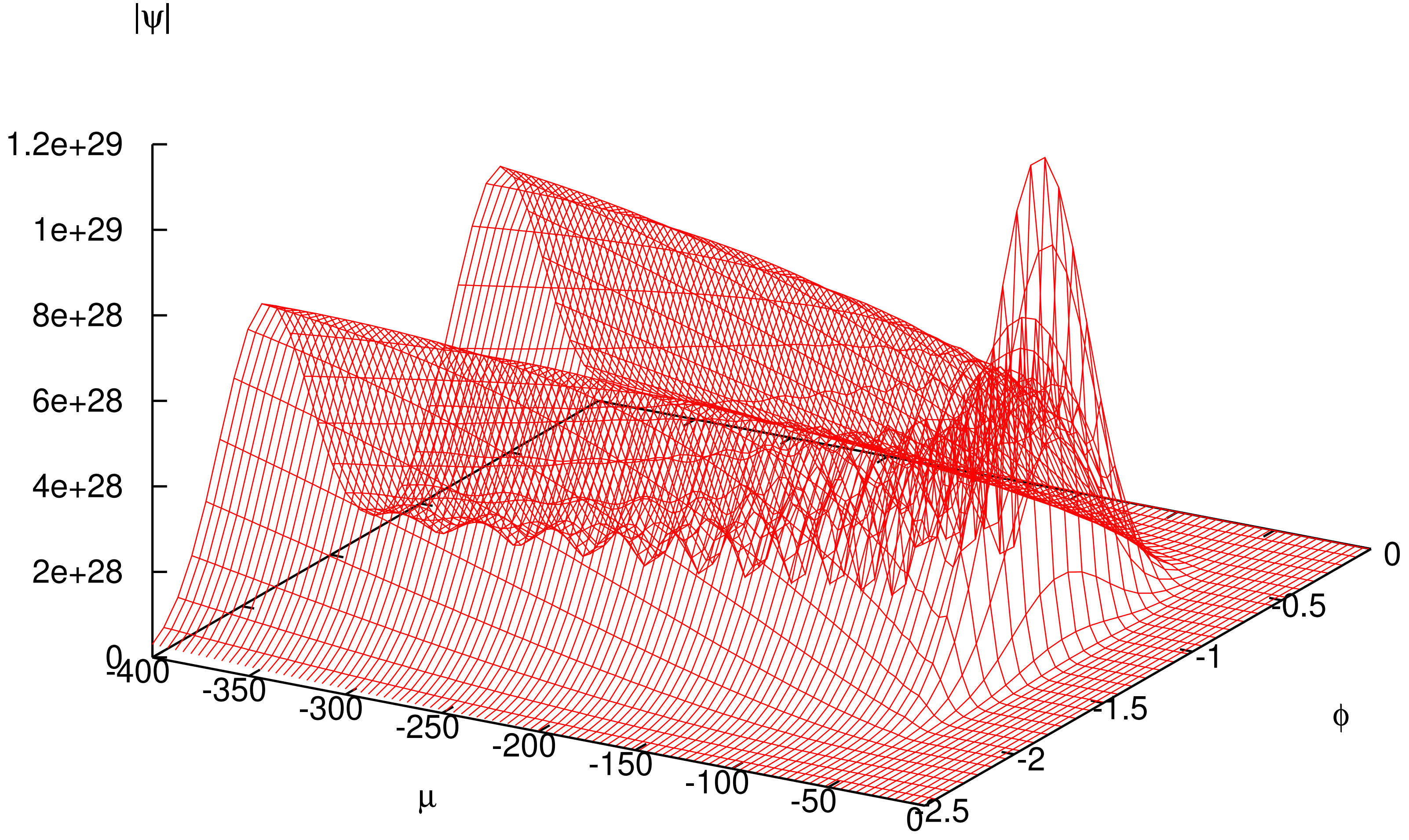
$$\psi(\mu, \Pi) = \mu^{1/4} \exp(i\sqrt{3/16\pi}\sqrt{\Pi^2 - \pi}\ln\mu) \exp((\Pi - \Pi_0)^2/2\sigma^2)$$

  where σ - spread in Π.
- After calculation of $\psi(\mu, \Pi)$ we obtain $\psi(\mu, \Pi)$ via Fourier transform.
- The resulting solution may be asymmetric, but as Θ commutes with parity reflection operator ψ may be symmetrized after calculation.

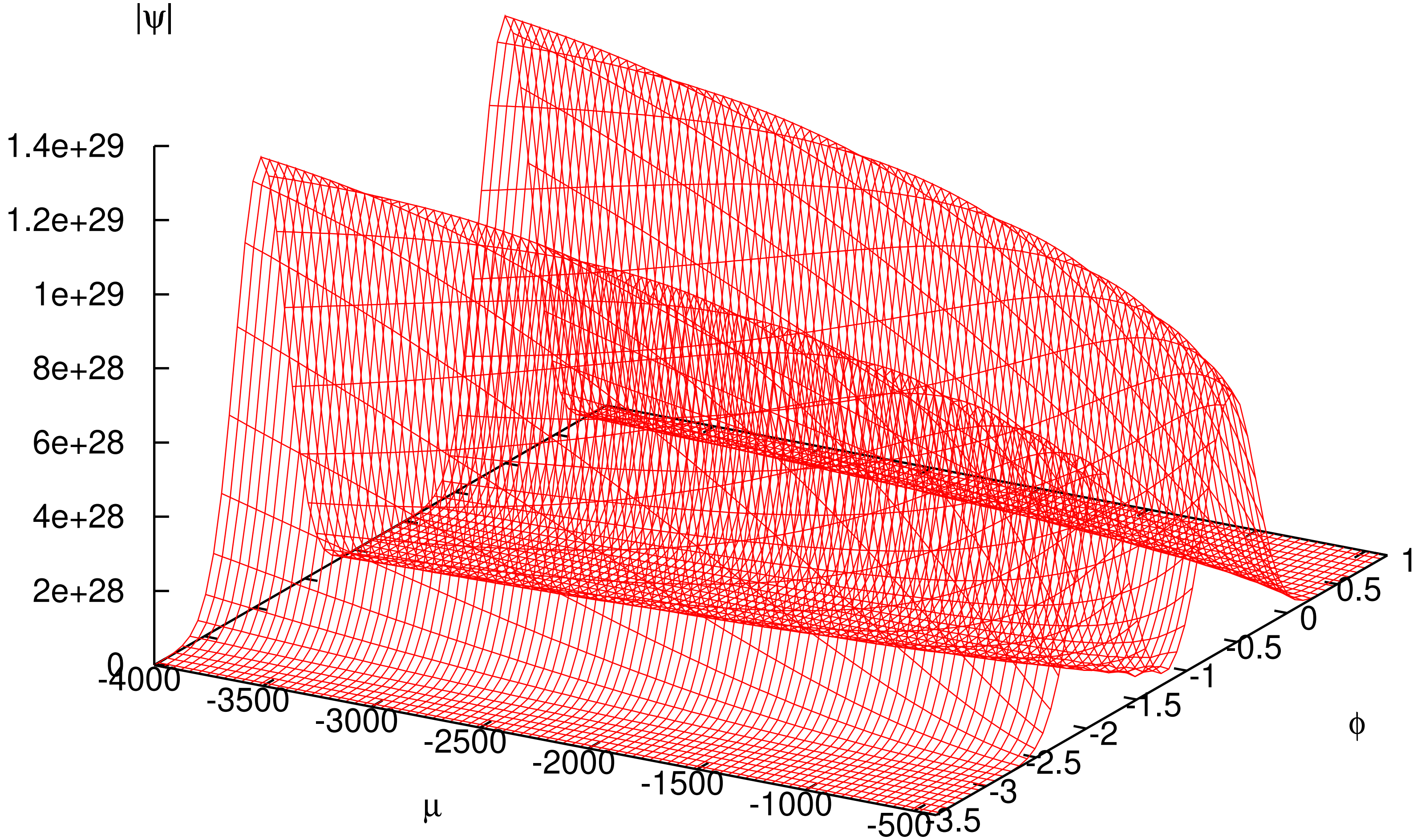
Evolution for positive μ



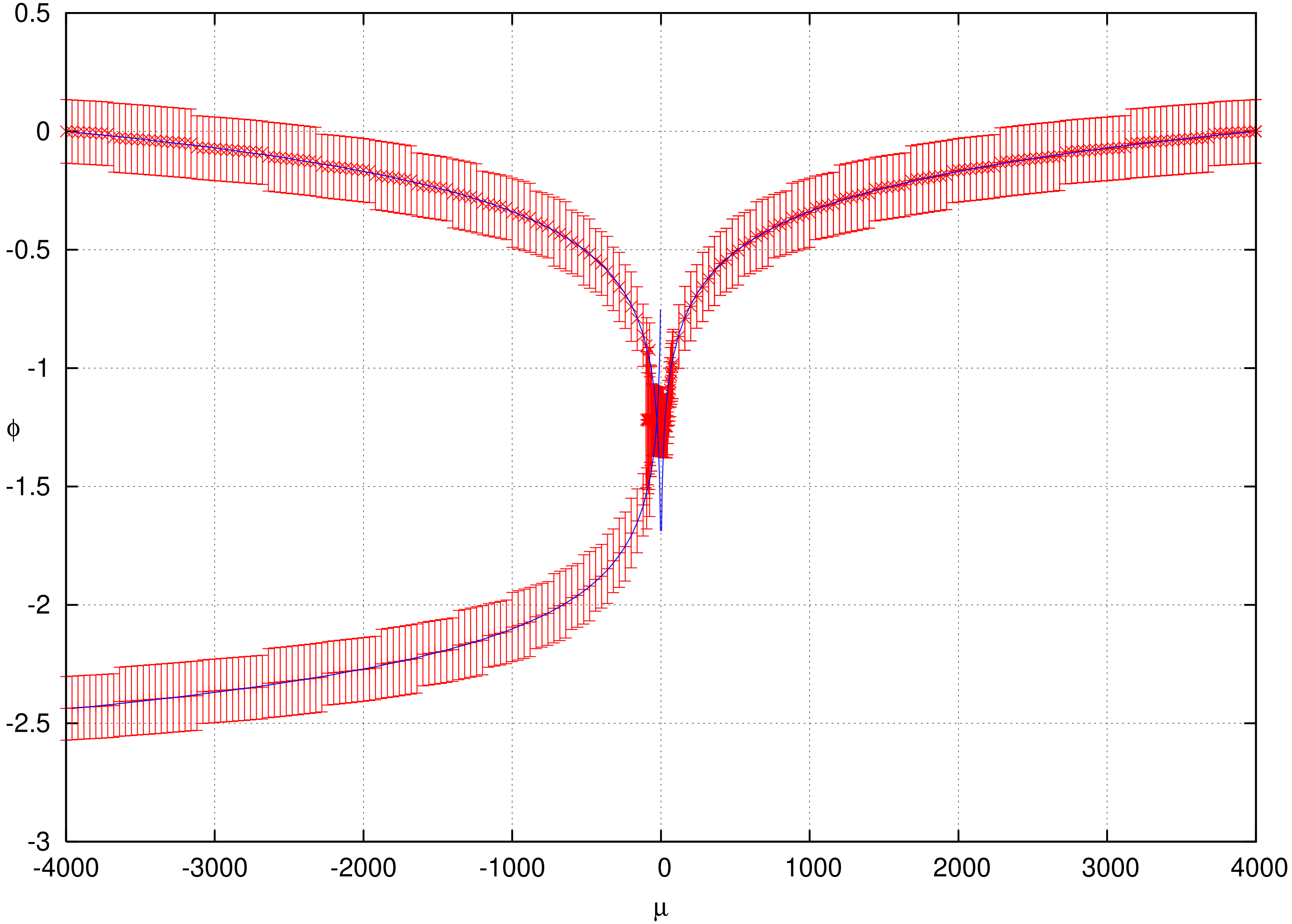
Evolution: small negative μ



Evolution: large negative μ



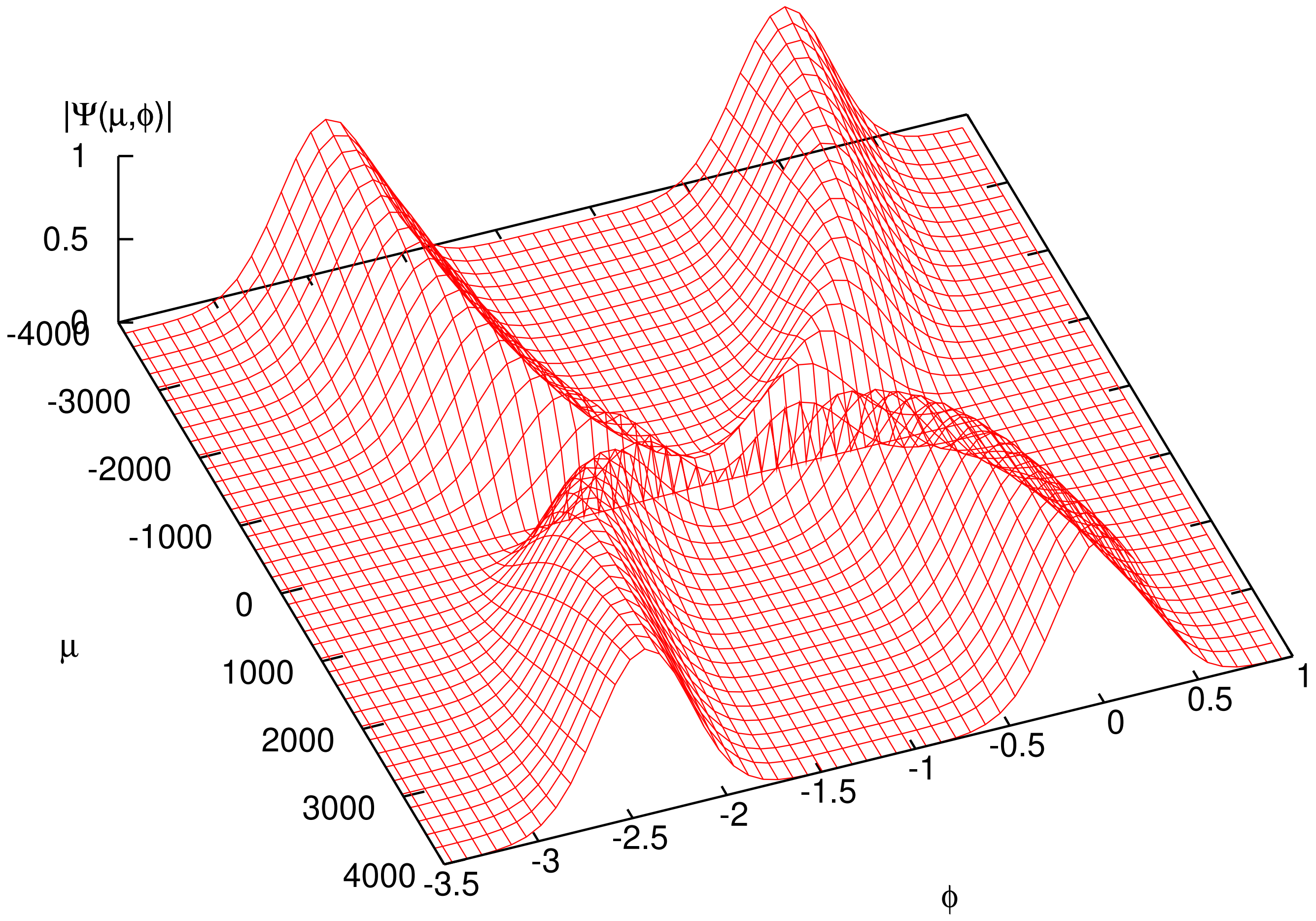
Expectation values of $\widehat{\phi_\mu}$ *vs* $\phi_{\rm class}(\mu)$



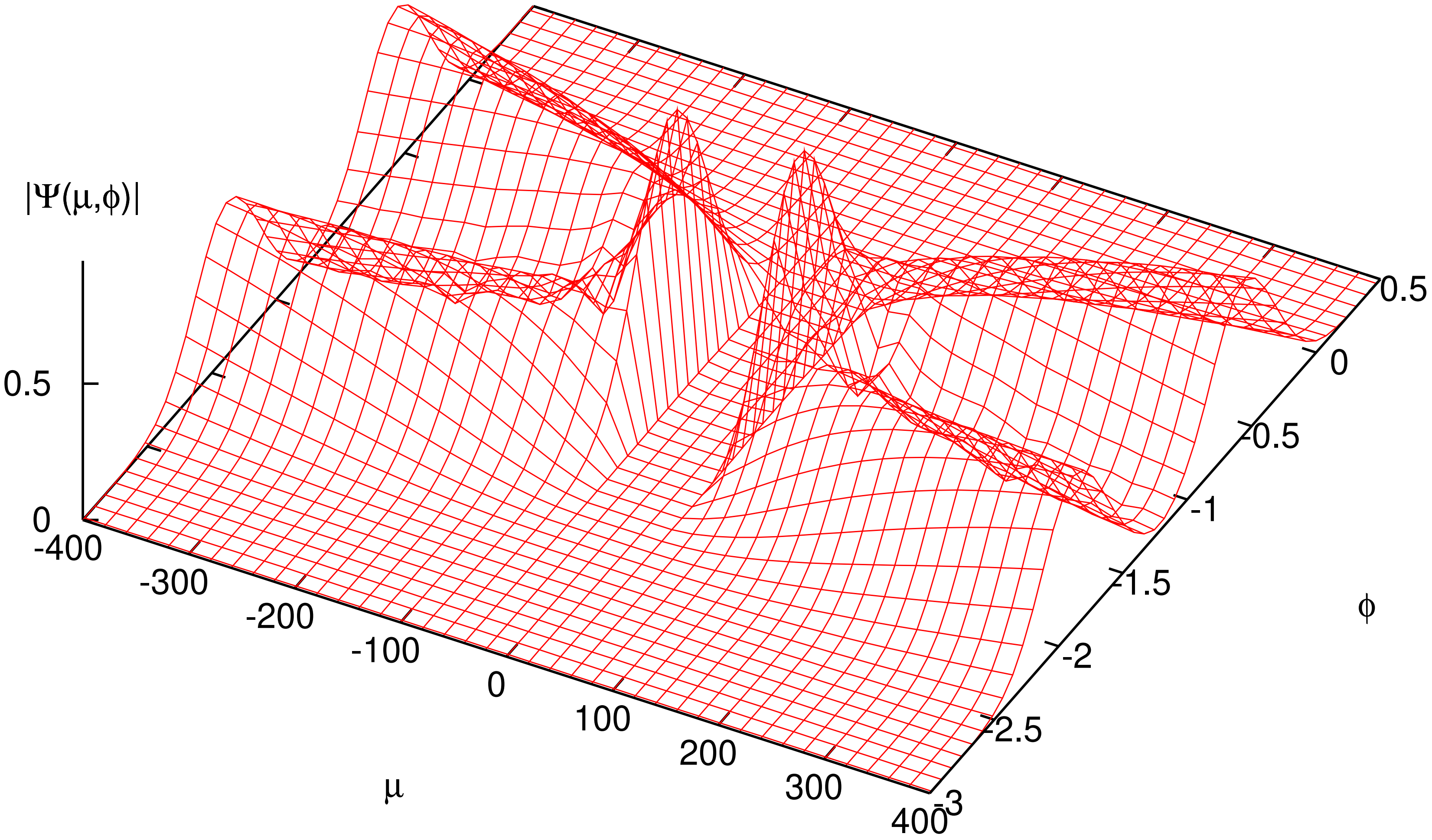
μ-IVP - the results

- The state remains coherent during entire evolution (constant spread).
- For both orientations ($\mathrm{sgn}(\mu)$) ψ well approximates some WdW solutions except small $|\mu|$ region. (result for $+$ve μ: Singh, Vandersloot: gr-qc/0507029)
- After crossing the singularity ψ is hugely amplified (order $\exp(\Pi_0)$).
- For negative μ the state splits onto two wave packets: a contracting and an expanding branch (with almost equal amplitude).
- The expectation values $\widehat{\phi_\mu}$ for expanding and contracting wave packets agree with classical trajectories $\phi_{\mathrm{class}}(\mu)$.
- Due to amplification only negative part relevant for symmetrization.

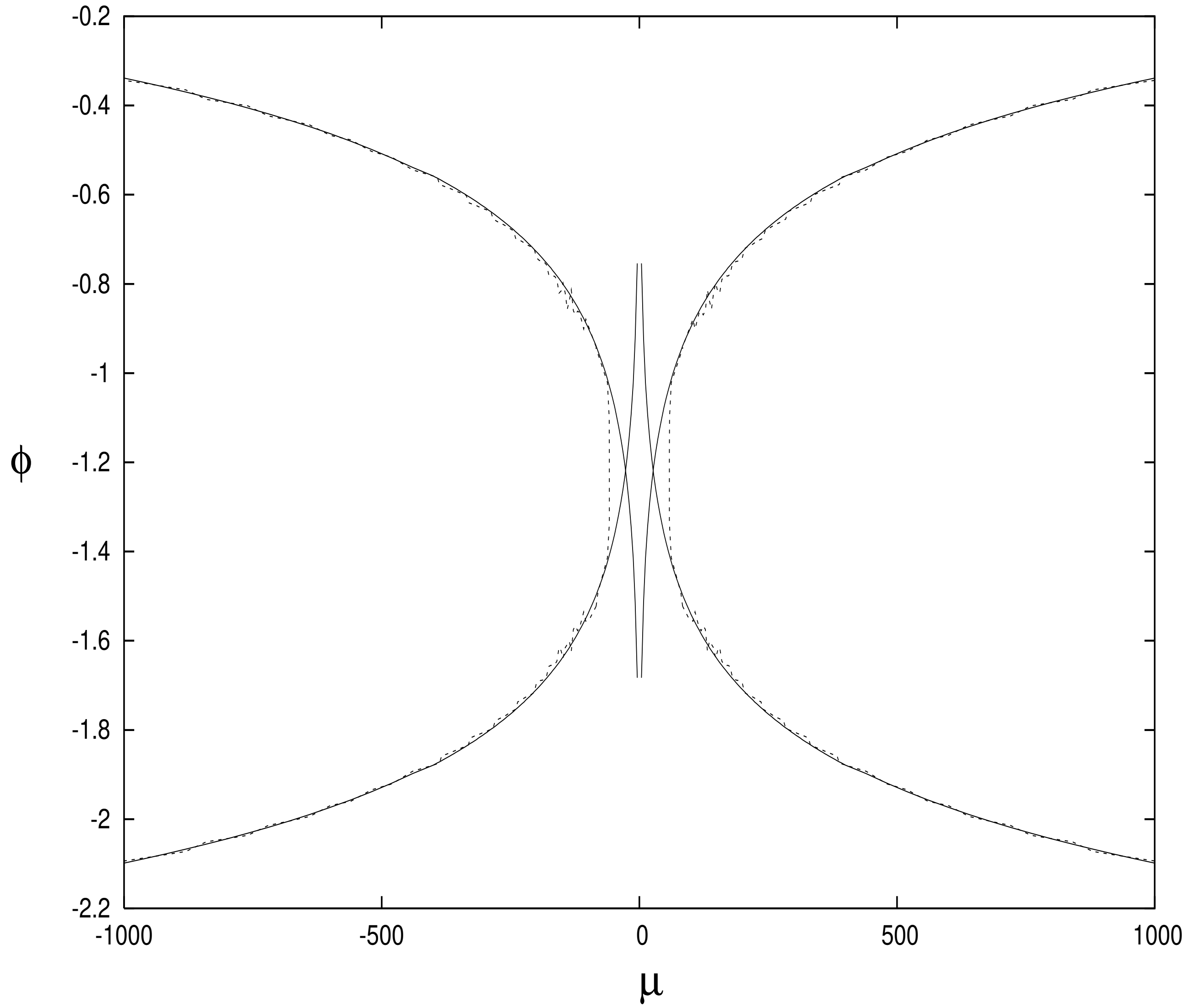
Symmetrized ψ



Symmetrized ψ *near* $\mu = 0$



Classical and quantum trajectories of ϕ



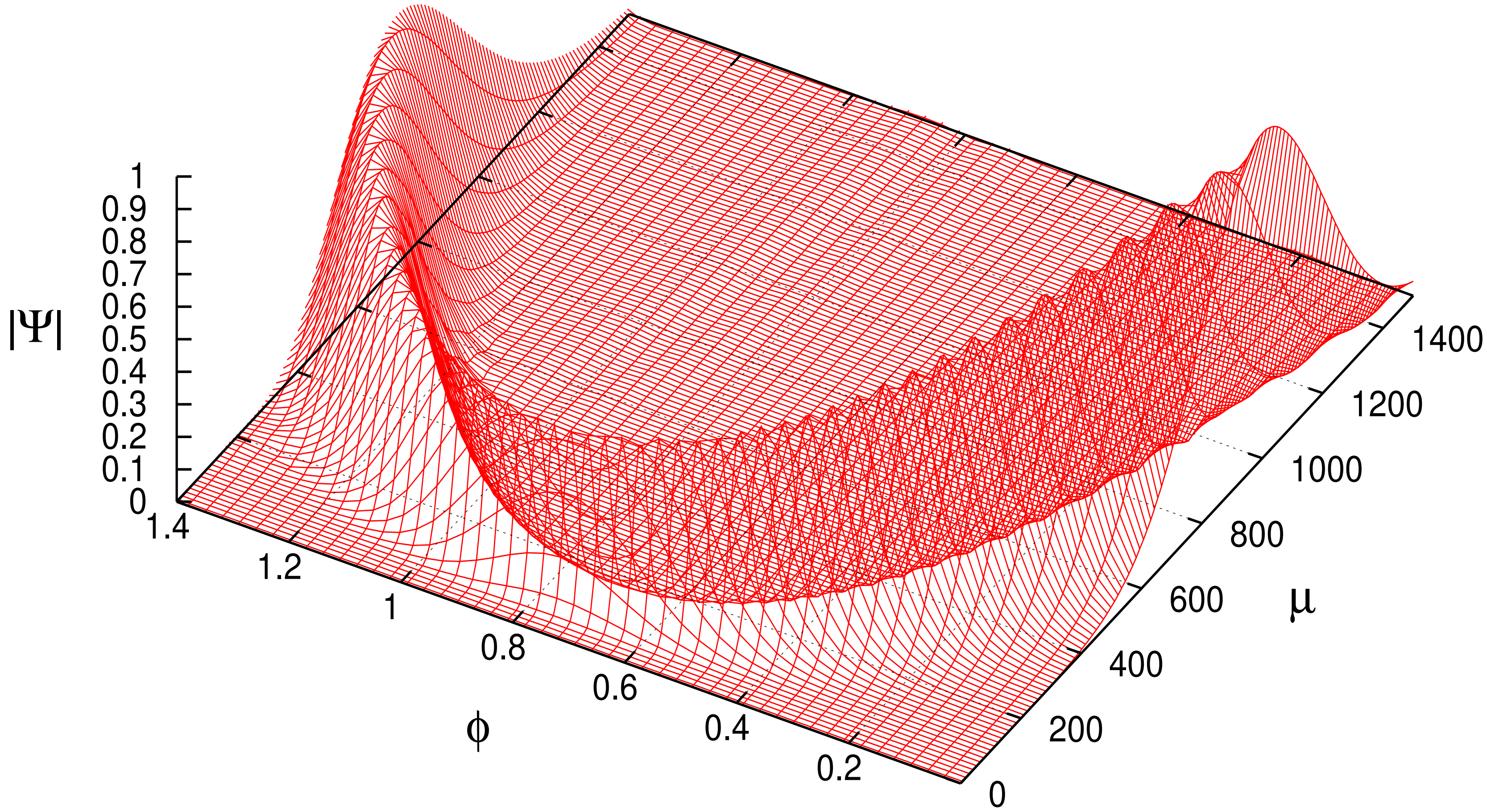
Symmetrization: the results

- The resulting evolution agrees closely with WdW (and classical) up to small $|\mu|$ region, where the quantum dynamics causes bounce in scale factor.
- Thus the quantum geometry effects make gravity repulsive for very small μ.
- Classical expanding universe is connected with a classical contracting one through a quantum gravitational bridge.

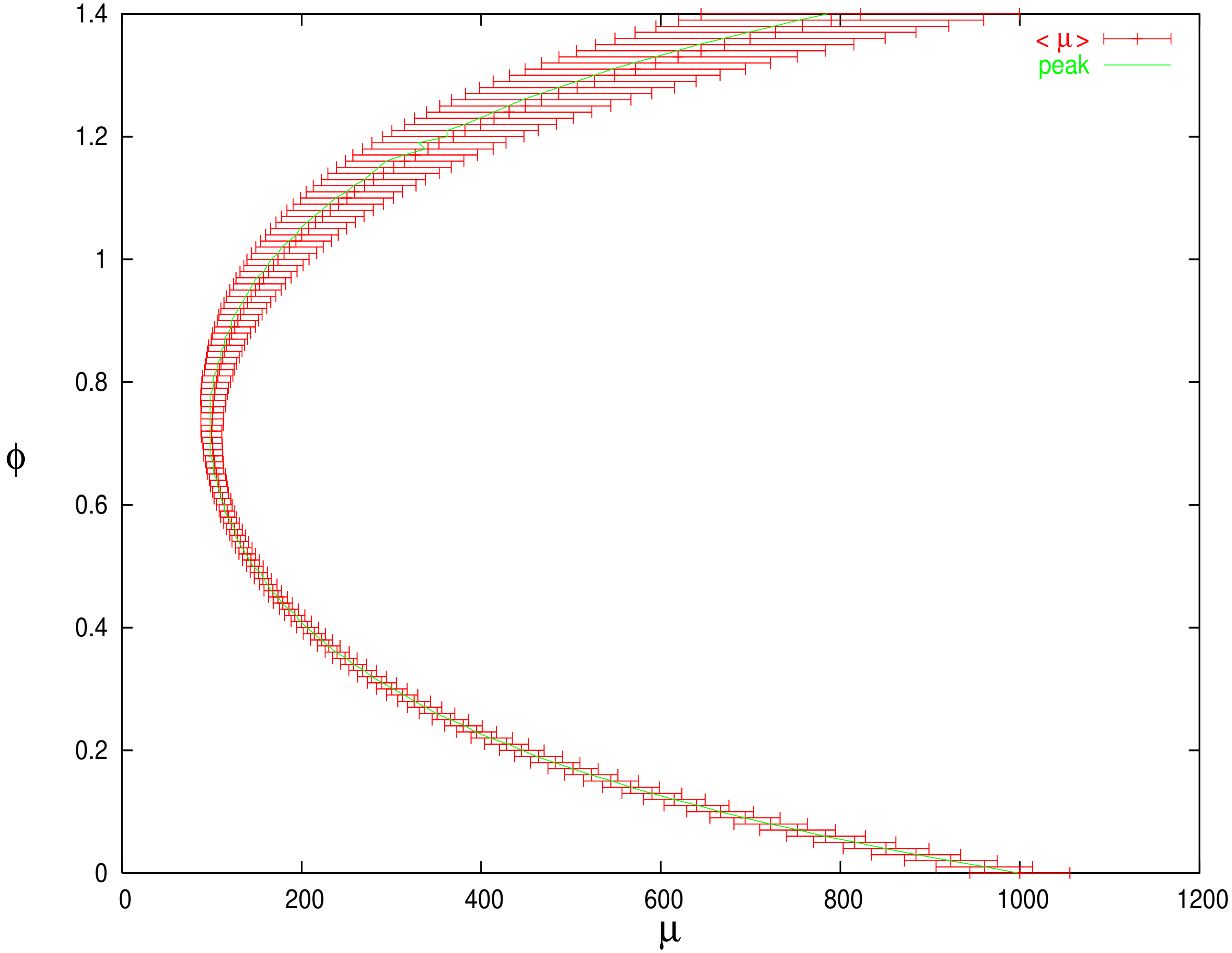
ϕ-IVP

- The wave function $\psi(\mu, \phi)$ is calculated directly.
- Initial conditions $(\psi, \partial_\phi \psi)$ are specified at $\phi = \phi_0$ as follows:
  - $\psi(\mu, \phi_0)$ is a Gaussian peaked around chosen large μ.
  - $\partial_\phi \psi(\mu, \phi)|_{\phi_0}$ is specified by classical equations of motion.
- The symmetry of ψ with respect reflection in μ is imposed at the level of initial conditions.

$\psi(\mu, \phi)$ *for* ϕ*-IVP*



$$\langle \psi | \widehat{\mu_\phi} | \psi \rangle$$



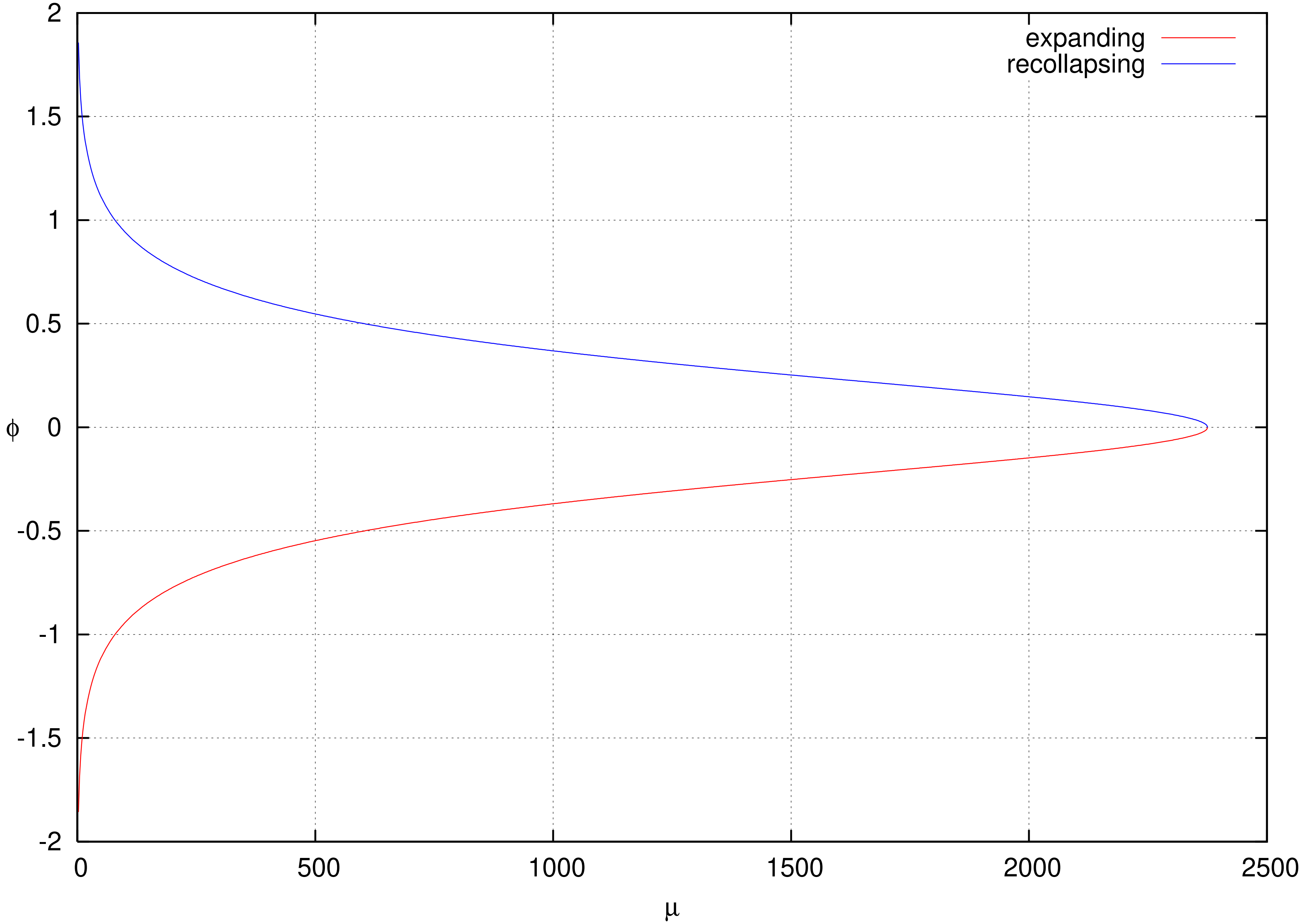
ϕ-IVP: the results

- The wave packet follows the WdW solution till small $|\mu|$ and bounces in scale factor near $\mu = 0$ (confirmation of the μ-IVP results).
- The spread of the profile $\psi(\mu, \phi = \mathrm{const})$ is small at the bounce point. Possibility of existence of the classical dynamics (modification of FRW one) well approximating the entire evolution of the universe (talk by K. Vandersloot).

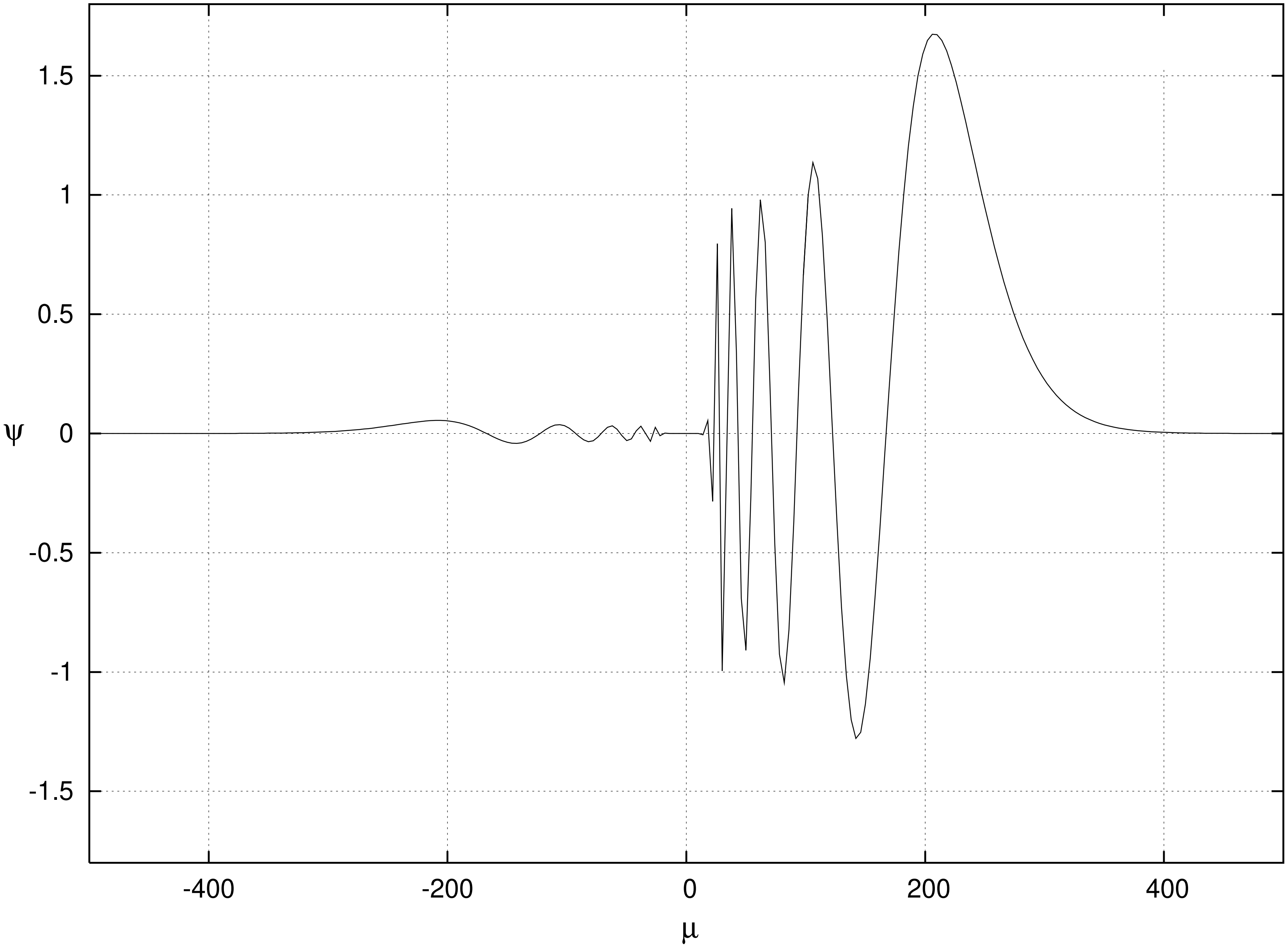
Closed (spherical) universe

- We consider isotropic closed universe with massless scalar field (QUANTIZATION WITHIN LQC FRAMEWORK: M. BOJOWALD, K. VANDERSLOOT GR-QC/0303072).
- Gravitational Hamiltonian (so Θ) still of the form
$$\widehat{H}_{\rm grav}\,|\mu\rangle = H^{+}(\mu)\,|\mu+4\mu_0\rangle + H^{0}(\mu)\,|\mu\rangle + H^{-}(\mu)\,|\mu-4\mu_0\rangle \ ,$$
but $H^0(\mu)$ contains additional term corresponding to global curvature.
- The (eventual) semiclassical state $\psi(\mu,\Pi)$ has to be composed of the eigenstates of Θ which decay beyond maximal size of the universe.
- As closed universe contains expanding and recollapsing regions ψ has to describe both of them.
- $\phi_{\rm class}$ takes two values for given μ, so each eigenstate has to be combination (with equal amplitudes) of expanding and contracting 'plain waves'.

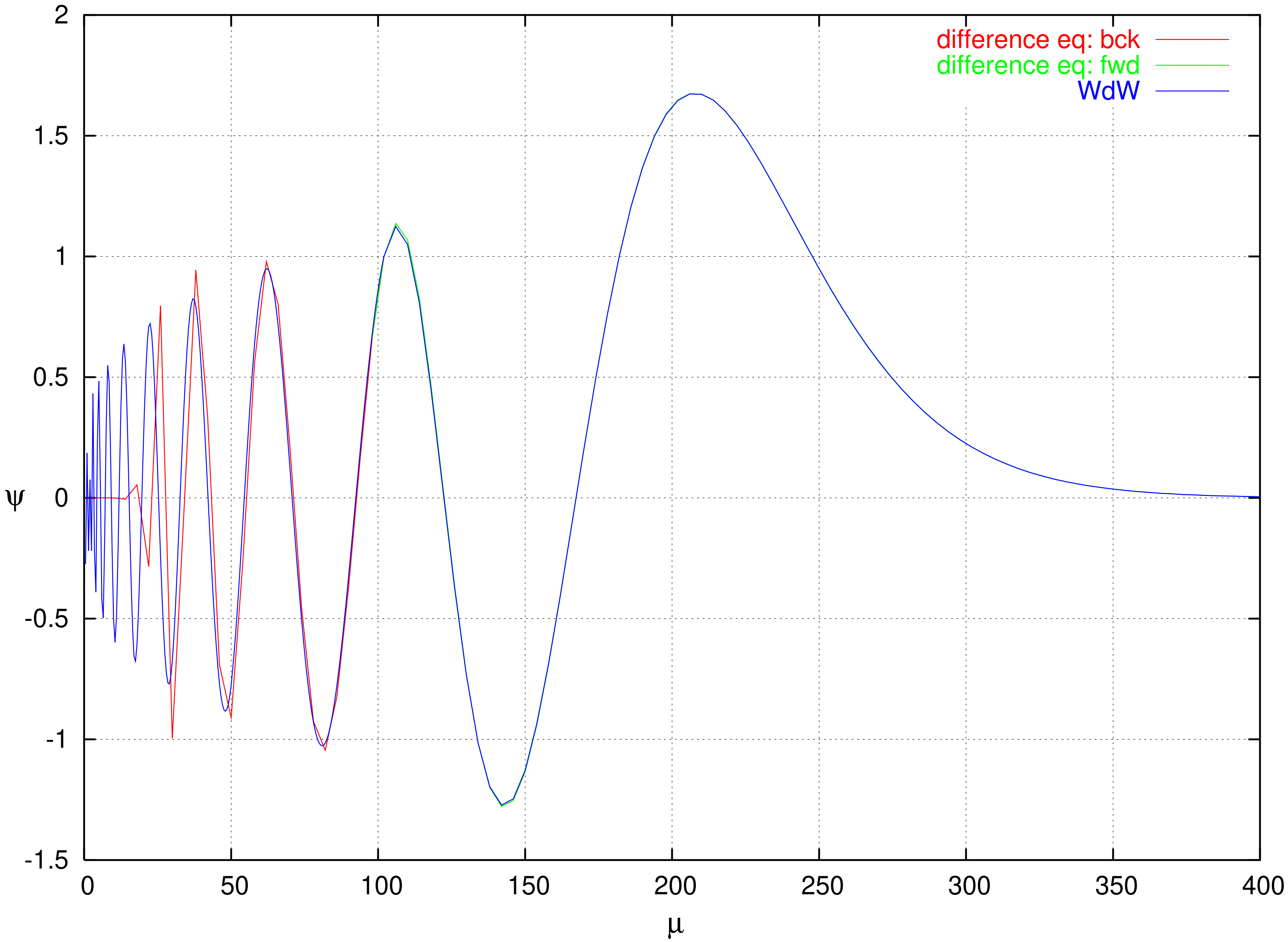
Classical trajectory



Converging eigenstate



Comparison with WdW



Closed universe: the results

- The generic eigenfunction of Θ blow-up exponentially beyond maximal scale factor, but in each eigenspace corresponding to eigenvalue $\lambda = \Pi^2 = k^2 + m^2$ (m-fixed numerical constant) there exist functions exponentially decaying for $a > a_{\max}$.
- The 'good' eigenfunctions asymptotically approach the WdW solutions (good semiclassical limit). So the semiclassical coherent state can be composed.
- Problem: We may expect bounce near singularity, but specifying the initial data in μ-IVP we impose its presence/absence 'by hands'. Thus to determine proper composition of coherent state we need to check the existence of bounce (using ϕ-IVP).

Summary

- Flat universe:
  - The contracting universe would bounce into an expanding one before hitting the 'big crunch' singularity.
  - Thus an expanding universe is connected to vast classical contracting universe through quantum gravitational effects.
  - The result has been verified by different numerical approaches.
- Closed universe:
  - There exist eigenstates with proper large scale behavior (decay for $a > a_{\max}$, approach WdW solutions). The semiclassical coherent states can be constructed.
  - Open issue: exact composition of $\psi(\mu, \pi)$ requires knowledge whether recollapsing universe bounces (need for verification via ϕ-IVP).