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ON THE REGULARIZATION AMBIGUITIES OF LOOP QUANTUM GRAVITY

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- → FINITENESS OF THE QUANTUM HAM. CONST. IN LQG.
- → REGULARIZATION AMBIGUITIES (THE m-AMBIGUITY).
- → THE ULTRA-VIOLET PROBLEM AND AMBIGUITIES IN QFT.
- → REVISITING THE UV PROBLEM IN LQG.
- → A SOLUTION OF THE PUZZLE? (RESULTS IN 2+1 AND 3+1 DIMENSIONS)
- → CONCLUSIONS

FINITENESS IN LQG:

$$B^a_k = \epsilon^{abc} F^k_{bc}$$

$$H = \frac{(\vec{E}_i \times \vec{E}_j)\cdot \vec{B}_k(A)\, \epsilon^{ijk}}{\sqrt{\vec{E}_l \cdot (\vec{E}_m \times \vec{E}_n)\, \epsilon^{lmn}}} \qquad \hat{H} = \frac{-i}{\hbar}[\hat{V}, \hat{\vec{A}}_i]\hat{\vec{B}}_i(A)$$

Thiemann

$$\{V, A\} \sim \frac{\vec{E} \times \vec{E}}{\sqrt{\vec{E}\cdot(\vec{E}\times\vec{E})}}$$

- $\hat{V}$ quantum volume op. which is well defined i.e., FREE OF UV PROBLEMS

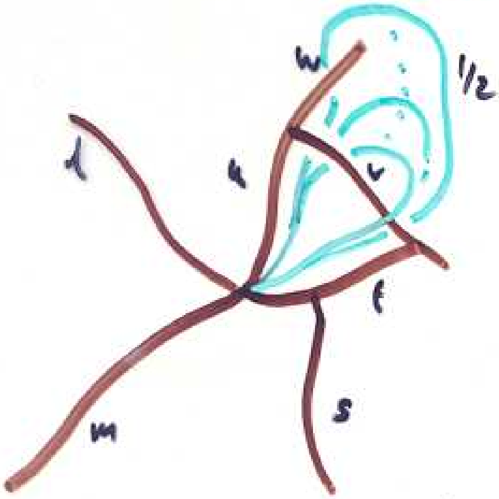
- $\vec{A}_i \cdot \vec{B}_i(A)$ must be regularized. This is done by means of a natural "point-splitting"

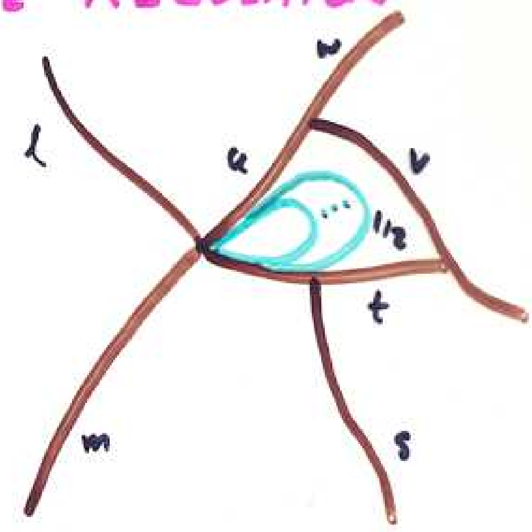
γ  $\vec{t}$, $\vec{s}$

$$U_\gamma(A) = 1 + \epsilon^2 \vec{B}_i(A)\cdot(\vec{s}\times\vec{t})\,\tau_i + \cdots$$

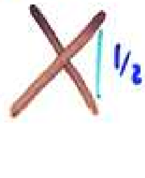
HOLONOMY AROUND γ

REMOVING THE REGULATOR

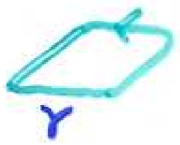




NO ULTRAVIOLET PROBLEM!

$\hat{H}$ ▷  =  +  + ...

THE AMBIGUITY PROBLEM

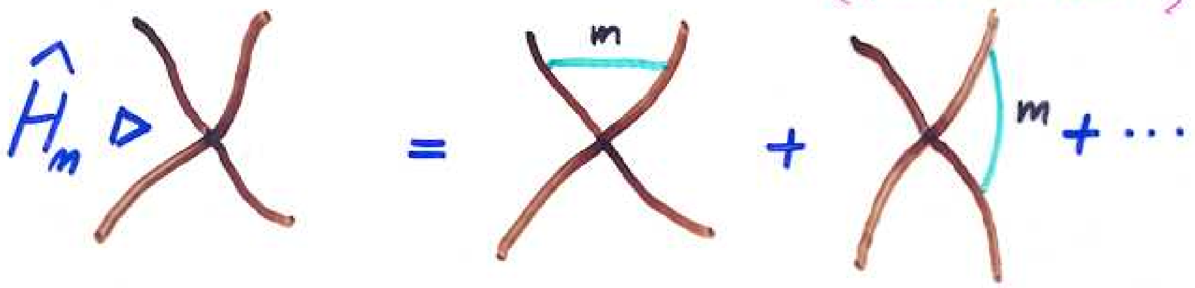


$$\overset{m}{\pi}(U_\gamma(A)) = \mathbb{1}^{(m)} + \epsilon^2 F_{ab}^{\ i} s^a t^b \tau_i^{(m)} + \dots$$

UNITARY REPRESENTATION OF SPIN "m"

GENERATOR IN THE i-TH DIRECTION

FOR EACH $m \in \mathbb{Z}/2$ THERE IS A CONSISTENT QUANTIZATION OF THE HAMILTONIAN CONST.
(Rovelli - Gaul)



THE SPACE OF A PRIORI CONSISTENT QUANT. IS INFINITE-DIMENSIONAL !

$$\hat{H} = \sum_m a_m \hat{H}_m \qquad \sum_m a_m = 1$$

ARE ALL THESE THEORIES PHYSICALLY DIFFERENT?

WE SHOULD HOPE THAT THIS IS NOT THE CASE IF LOOP QUATUM GRAVITY IS TO BE THOUGHT OF AS A FUNDAMENTAL THEORY.

EVIDENCE FROM LOOP QUANTUM COSMOLOGY ④

- SPACE OF SOLUTIONS OF THE HAMILTONIAN CONSTRAINT DEPENDS ON "m"! Kevin VANDERSLOOT Phys. Rev. D.

$$\text{ORDER OF THE DISCRETE EQ.} = 8m$$

- Effective HAMILTONIAN OF LQC: the quantum corrections computed in the literature are "m" dependent

J. Willis Ph.D. Th.

$$H_{eff} = H_{CLASS} + \sum_n l_p^{2n} c_n(m)\, \delta H_n$$

THESE COEFFICIENTS DO DEPEND ON m (K. VANDERSLOOT)

- IN SIMPLE MODELS WHERE THERE IS COMPLETE CONTROL OF THE PHYSICAL INNER PRODUCT ONE CAN SEE THAT PHYSICAL PREDICTIONS ARE UNDOUBTLY REGULARIZATION DEPENDENT! FROM RESULTS OF K. NOUI, K. VANDERSLOOT, A.P.

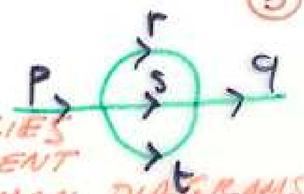
THE REPRESENTATION AMBIGUITY HAS GENUINE PHYSICAL EFFECTS IN LOOP QUANTUM COSMOLOGY

$$\Downarrow$$

THERE IS AN INFINITE DIMENSIONAL SET OF QUANTUM COSMOLOGIES!

THE UV PROBLEM IN QFT e.g. ϕ^4

ITS ORIGIN IS IN THE MATHEMATICAL DIFFICULTIES ASSOCIATED TO MAKING SENSE OF PROD. OF DIST. AT THE SAME POINT $\Rightarrow$ IN PERT. THEORY IMPLIES UV DIVEGENT FEYNMAN DIAGRAMS



STANDARD REGULARIZATION + COUNTER TERMS

DIM. REGULARIZATION
UV CUT-OFF
PAULI-VILAR REG.

MORE CAREFUL TREATMENTS

EPSTEIN-GLASER
Scharf "FINITE QED"

AMBIGUITIES

MUST BE FIXED BY
RENORMALIZATION CONDITIONS

NON-RENORMALIZABLE THEORIES

RENORMALIZABLE THEORIES

PERTURBATIVE QUANTUM GRAVITY

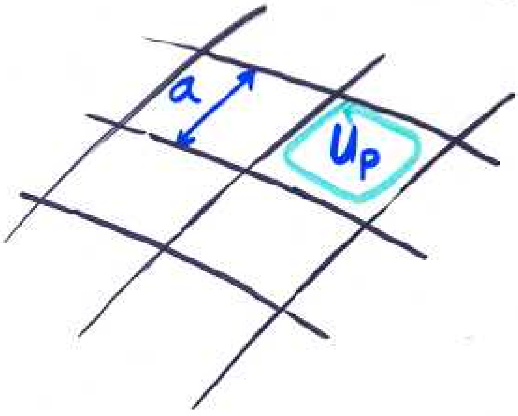
$$\mathcal{L}_{eff} = \mathcal{L}_{CLASS} + \sum_n l_p^{2n} c_n \, \delta\mathcal{L}_n$$

LITTLE PREDICTIVE POWER

ARE WE NOT IN THE SAME SITUATION IN LOOP QUANTUM GRAVITY?

REGULARIZATION AND AMBIGUITIES IN NON-PERTURB. QFT: LATTICE GAUGE THEORY

$$S^{(1/2)}_{LYM} \simeq \frac{1}{g^2}\sum_p \left(1-\frac{1}{4}\,\mathrm{Tr}\left(U_p+U_p^\dagger\right)\right)$$



$$S^{(m)}_{LYM} \simeq \frac{1}{g^2}\sum_p \left(1-\frac{\mathrm{Tr}\left(\overset{m}{\Pi}(U_p)+\overset{m}{\Pi}(U_p^\dagger)\right)}{2(2m+1)}\right)$$

OR MORE GENERALLY $S_{LYM} = \sum_m a_m\, S^{(m)}_{LYM}$

THE PREVIOUS ACTIONS ALL APPROXIMATE THE STANDARD YANG-MILLS ACTION WHEN $a \to 0$ FOR FIXED SMOOTH FIELD CONFIGURATIONS

("NAIVE CONTINUUM LIMIT")

THE RENORMALIZABILITY OF YANG-MILLS THEORY IS WHAT GRANTS THE USE OF THE PREVIOUS ACTIONS AS APPROXIMATIONS OF QUANTUM YANG-MILLS.

"LOW ENERGY LIMIT" $S_{eff} = S_{YM} +$ "DIMENSION FIVE AND HIGHER OPERATORS"

BACK TO LOOP QUANTUM GRAVITY

$$\hat{H} = \sum_m q_m \, \hat{H}^{(m)}$$

REGULARIZATION OF H USING HOLONOMIES IN THE SPIN m REPRESENT.

A) RESULTS IN LOOP QUANTUM COSMOLOGY

- HIGHER CURVATURE CORRECTIONS IN THE EFF. HAMILT. DEPEND ON THE REGULARIZATION CHOICE!
- PHYSICAL INNER PRODUCT DEPENDS ON REGULARIZATION

B) THINKING ALONG THE LINES OF LATTICE G. THEORY

$$S_{\substack{\text{GRAV.}\\\text{EFF.}}} = S_{\substack{\text{EINST.}\\\text{HILBERT}}} + \text{"HIGHER CURVATURE CORRECTIONS"}$$

QUANTUM GRAVITY MIXED UP WITH REGULARIZATION DEPENDENCE

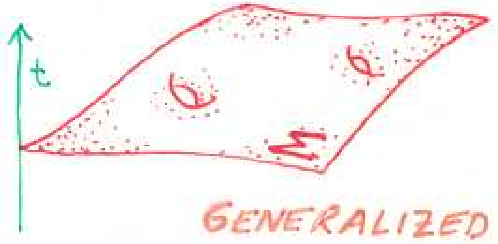
BUT!

A) LOOP QUANTUM COSMOLOGY IS A PHENOMENOLOGICAL DESCRIPTION INSPIRED BY LQG AND NOT A FUND. THEORY.

B) THE LOW ENERGY LIMIT OF A GENERALLY COVARIANT THEORY IS EXPECTED TO BE A VERY DELICATE ISSUE.

CANONICAL CURVATURE CONSTRAINT

$$\hat{F}[N]|\Psi\rangle = \int_{\Sigma} \mathrm{tr}(N\,\hat{F}(A))|\Psi\rangle = 0 \quad \forall N$$

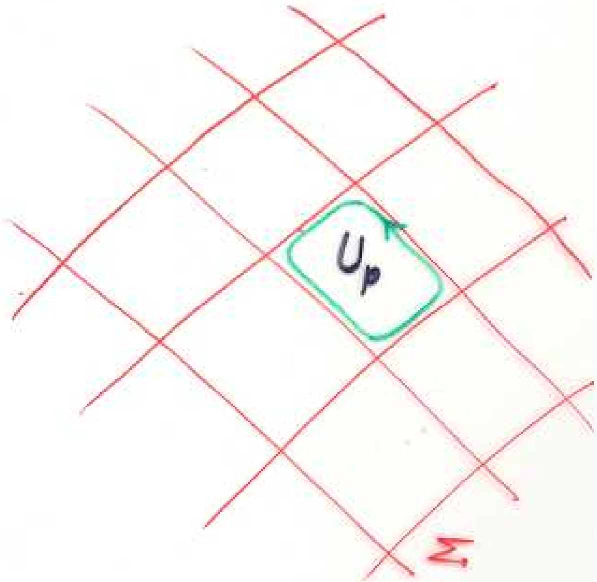


GENERALIZED PROJ. OPERATOR

$$P = \text{"}\delta(F)\text{"} \equiv \int DN\, e^{i\hat{F}[N]}$$

REGULARIZATION (Noui - AP)

$$\hat{F}^{(1/2)}_{\Delta}[N] = \sum_{p} \mathrm{Tr}(N_p \hat{U}_p)$$



THE m - AMBIGUITY

$$\hat{F}^{(m)}_{\Delta}[N] = \sum_{p} \frac{\mathrm{Tr}^{(m)}(N_p \overset{(m)}{\Pi}(U_p))}{C_m}$$

MORE GENERALLY

$$\hat{F}_{\Delta}[N] = \sum_{m} a_m \hat{F}^{(m)}_{\Delta}[N]$$

THERE IS AN INFINITE DIMENSIONAL SET OF REGULARIZATIONS SATISFYING THE "NAIVE CONTINUUM LIMIT"

HOWEVER THESE REGULARIZATIONS MAY DIFFER IN TERMS OF THEIR SOLUTION SPACE

A U(1) ANALOGY:

$$\overset{m}{\Pi}(U_p) \longrightarrow e^{im\phi_p}$$

$$\overset{m}{\Pi}(U_p) = \mathbb{1}^{(m)} \longrightarrow e^{im\phi_p} = 1$$

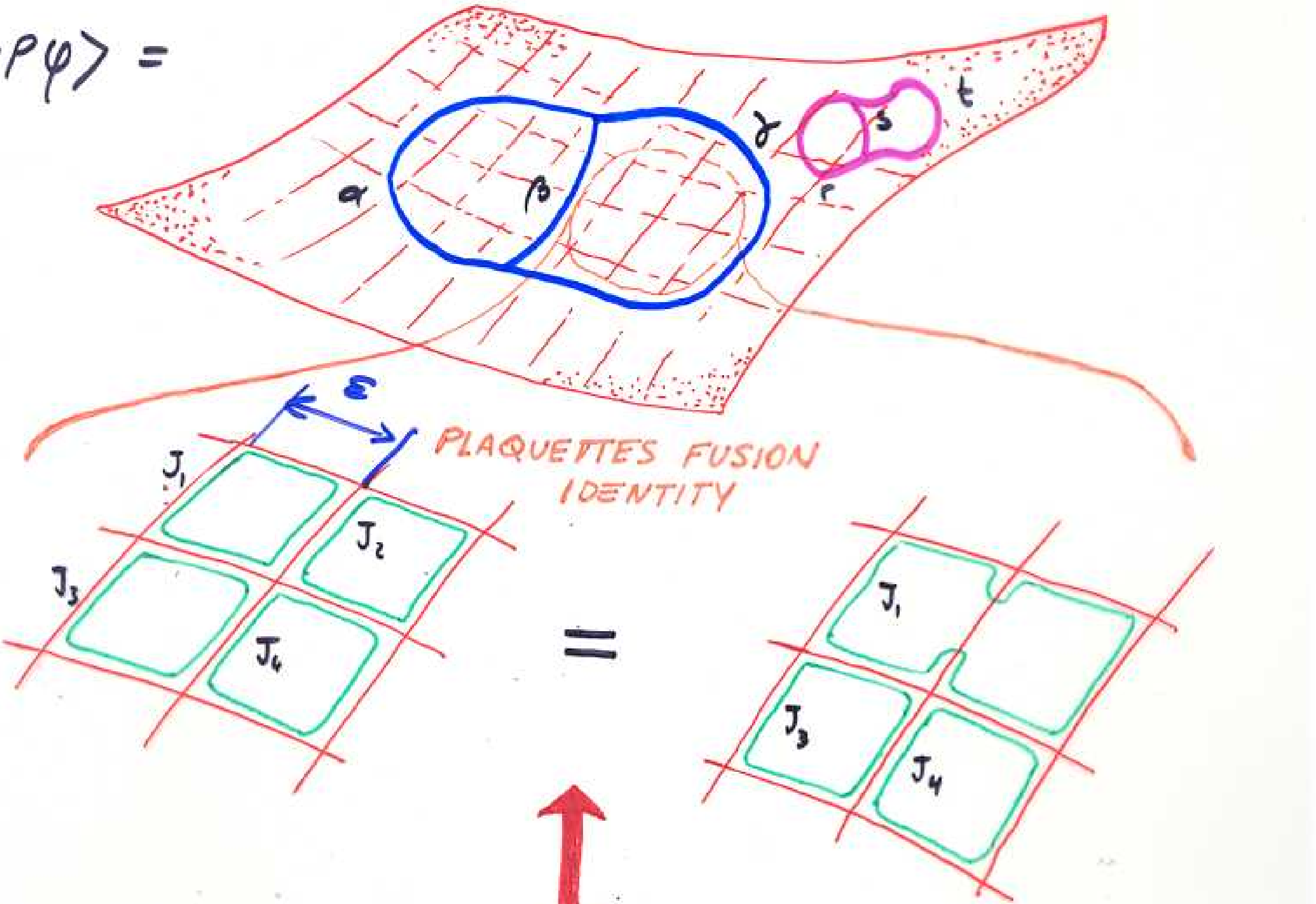
THE NUMBER OF SPURIOUS SOLUTIONS GROW WITH "m"!

REMOVING THE REGULATOR

$$P = \lim_{\epsilon \to 0} \sum_{J_p} \prod_p A_p(J_p)\, \hat{\chi}_{J_p}(U_p)$$

REGULARIZATION DEPENDENT

$\langle \psi P \psi \rangle =$



Anomaly Freeness (Bojowald - Perez)

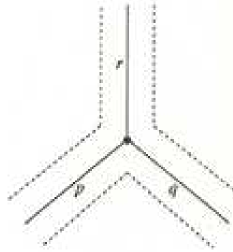
$$A_p(J) = \alpha\,(2J+1)$$

$$\hat{F}_\Delta[N] = \sum a_m\, \hat{F}^{(m)}_\Delta[N]$$ has no spurious solutions

ONLY FOR THESE REGULARIZATIONS ONE CAN REMOVE THE REGULATOR. THE PHYSICAL INNER PRODUCT AND HENCE THE QUANTUM THEORY TURNS OUT TO BE UNIQUE!

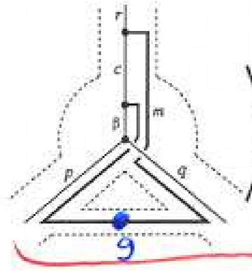
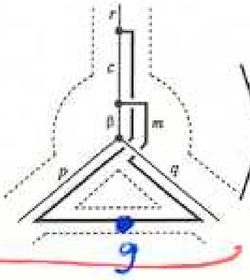
3+1 LOOP QUANTUM GRAVITY: SPURIOUS SOLUTIONS PRODUCED BY THE CHOICE OF REGULARIZATION

$$\underbrace{\hat{H}^{(m)}_{\Delta}|\text{vertex}\rangle = \frac{i}{\hbar}\left(\vec{B}_i(A)\cdot\left[\hat{V},\vec{A}_i\right]\right)|\text{vertex}\rangle}$$

$$\mathrm{Tr}\left(\left(\hat{h}^{(m)}[\alpha_{ij}] - \hat{h}^{(m)}[\alpha_{ji}]\right)\hat{h}^{(m)}[s_k]\,\hat{V}\,\hat{h}^{(m)}[s_k^{-1}]\right)\Big|$$  $\Big\rangle$

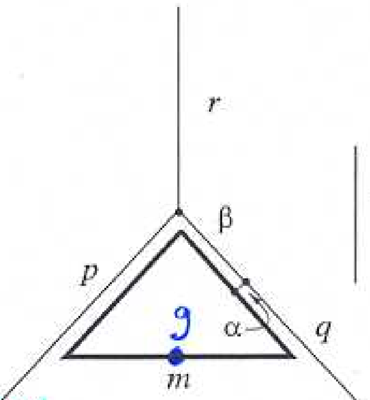
HOLONOMIES IN THE m-UNITARY REP

THE CHOICE OF m IS ARBITRARY.

$$= (-1)^{2m}\sum_{c\beta} V(p,q,m,c)_r^\beta \Bigg[$$  $\Big\rangle -$  $\Big\rangle\Bigg]$

THE RESULT OF THE ACTION OF THE REGULATED HAMILT. ON A VERTEX IS A SPIN-NETWORK STATE WHICH IS AN ODD FUNCTION OF THE HOLONOMY AROUND THE NEWLY CREATED LOOP! $(g \to g^{-1})$

EVEN FUNCTIONS ARE THEN SOLUTIONS

$$(\Psi| = \sum_{\phi\in\mathrm{Diff}[M]}\sum_{\alpha\beta} c^{\Psi}_{\alpha\beta}\Big\langle$$  $\Big|\ \mathrm{U}[\phi]\quad / \quad \{\alpha\} = even$

DIMENSION OF THE SPACE OF SOLUTIONS GROWS WITH "m". FOR INSTANCE $\alpha \in$ EVEN $\quad 0 \leq \alpha \leq 2m$

SPURIOUS SOLUTIONS!

CONCLUSIONS

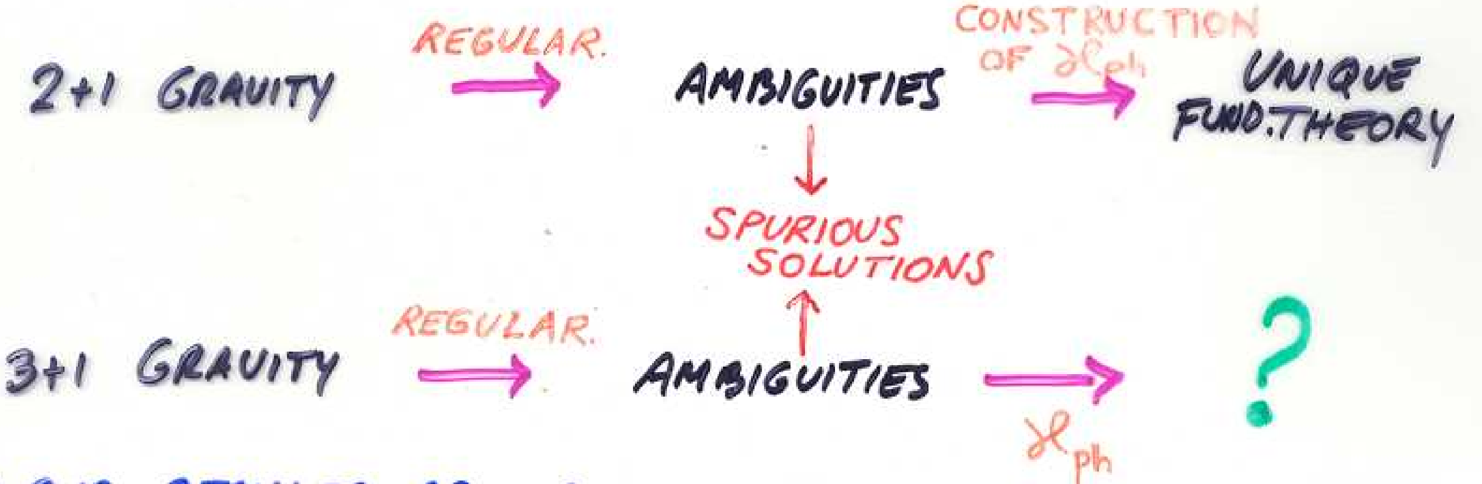
* FINITENESS OF THE HAMILTONIAN CONSTRAINT IS JUST ONE SIDE OF THE UV PROBLEM IN LOOP QUANTUM G.

* AS IN STANDARD QFT THERE ARE AMBIGUITIES ASSOCIATED TO THE REGULARIZATION PROCEDURE.

* THE AMBIGUITIES MIGHT HAVE SERIOUS CONSEQUENCES FOR THE PREDICTABILITY OF LOOP QUANTUM GRAV.

* CONTRARY TO WHAT IS OFTEN BELIEVED; FENOMENO_LOGICAL MODELS DON'T SEEM HELPFUL IN FIXING AMB. OF THE FULL THEORY.

* CONSISTENCY IN THE DEFINITION OF DYNAMICS MIGHT REDUCE THE APARENTLY LARGE DEGREE OF AMBIGUITY IN LOOP QUANTUM GRAVITY.



* OUR RESULTS ARE RELEVANT FOR OTHER APPROACHES TO DYNAMICS : CONSISTENT DISC. AND MASTER CONSI.P

GAMBINI - PULLIN

THIEMANN et al.

LOCAL EXITATIONS FOR $m=1$

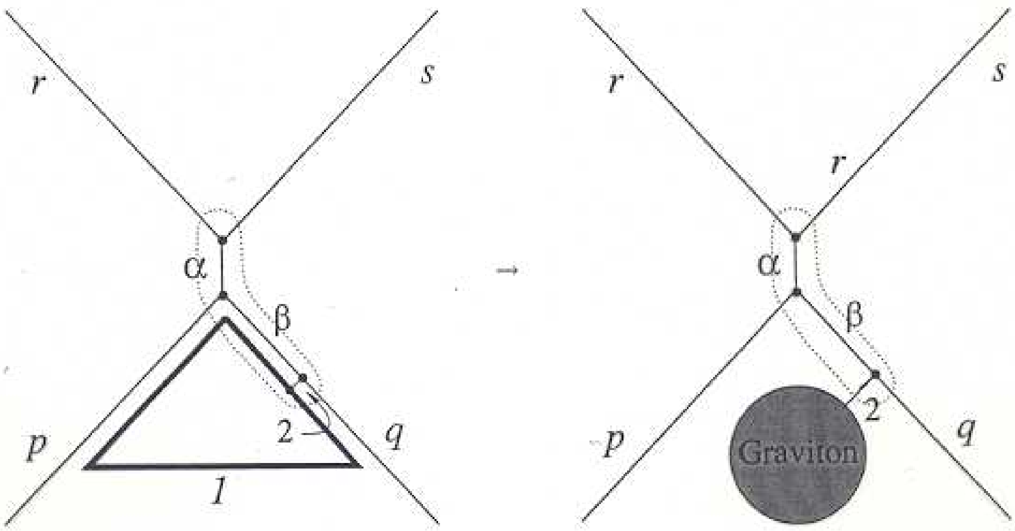


FIG. 1: Interpretation of the solutions (1) for $m = 1$ as graviton excitations. Starting from a trivial solution to the constraints given by a diff-invariant spin network with no exceptional edge we can construct a new solution as explained and illustrated here. The solution space is parametrized by the quantum numbers α and β in this figure. The dotted region corresponds to a single point in the spin network graph.

WE NEED MORE RELIABLE INSIGHTS !

- SEMICLASSICAL LIMIT OF LQG
  DIFFICULT!

- INTERNAL CONSISTENCY IN THE DEFINITION OF THE DYNAMICS : CONSTRUCTION OF THE PHYSICAL HILBERT SPACE. ✓

RESULTS

☆ 2+1 GRAVITY: AMBIGUITIES ARE PRESENT AT THE KINEMATICAL LEVEL BUT THE PHYSICAL HILBERT SPACE IS INDEPENDENT OF THEM : UNIQUE PHYSICAL THEORY.

→ BAD REGULARIZATIONS: INTRODUCE SPURIOUS SOLUTIONS OF CONSTRAINTS AND PRECLUDE THE ELIMINATIONS OF THE REGULATOR

→ GOOD REGULARIZATIONS: THE REGULATOR CAN BE REMOVED AND THE RESULTING THEORY IS UNIQUE (e.g. THE FUNDAMENTAL REP. OF SU(2))

☆ 3+1 GRAVITY: POSITIVE EVIDENCE

→ BAD REGULARIZATIONS: SPURIOUS SOLUTIONS

→ GOOD REGULARIZATIONS: THE FUNDAMENTAL REP. OF THE GAUGE GROUP.