ON THE QUANTUM ORIGIN OF THE SEEDS OF COSMIC STRUCTURE

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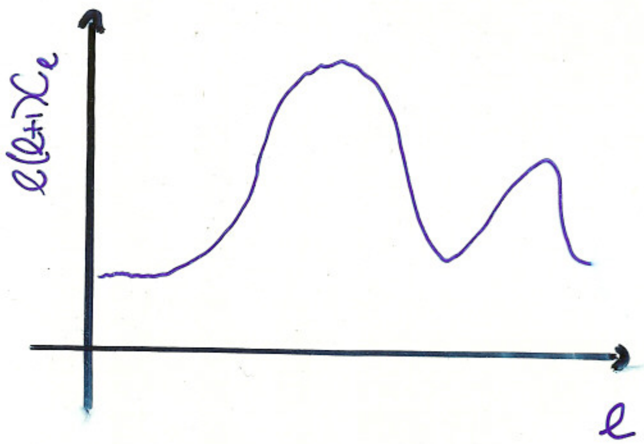
QUICK REMINDER: CMB, INFLATION



$$\frac{\delta T}{T} = \sum_{\ell} \sum_{m} c_{\ell m} Y_{\ell m}$$

$$C_\ell = \frac{1}{2\ell+1} \sum_{m} |c_{\ell m}|^2$$

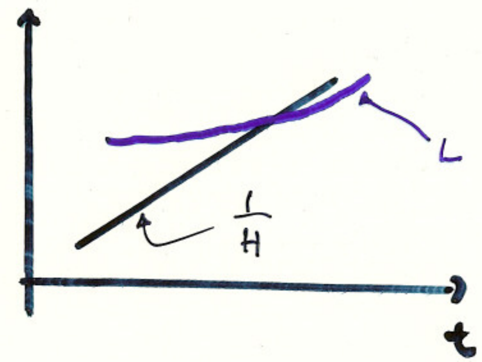
TELLS US ABOUT MATTER + GRAVITY AT LAST SCATTERING (SACHS-WOLFE EFFECT).

LEARNED A LOT ABOUT UNIVERSE:

- ALMOST PERFECTLY HOMOGENOUS
- INITIAL INHOMOGENEITIES OF THE SCALE FREE TYPE
- PRECISE DETERMINATION MANY COSMOLOGICAL PARAMETERS.
  EX.: UNIVERSE FLAT.

NOT SO EASY TO EXPLAIN WITH STANDARD COSMOLOGY :

- HORIZON PROBLEM
- FLATNESS PROBLEM



$$|\Omega - 1| \sim \frac{1}{(a')^2}$$

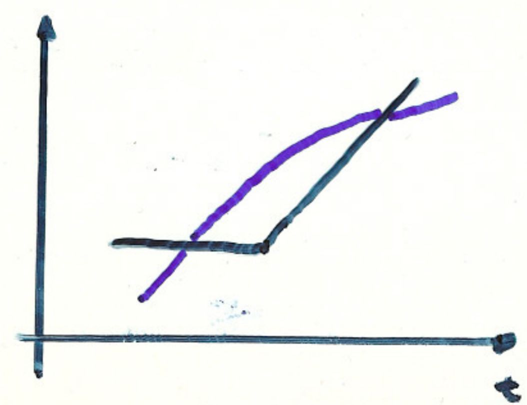
NEED SOME NON-STANDARD STUFF TO GET THIS, THOUGH!

IDEA TO SOLVE THEM : INFLATION

$a'' > 0$ AT SOME EARLY TIME

EXPLAINS FLATNESS
ALLEVIATES HORIZON PROBLEM

EVEN HAS SOMETHING TO SAY ABOUT THE INHOMOGENEITIES



INFLATION AND INHOMOGENEITIES

REMARKABLE COINCIDENCE OF SPECTRUM OF QM UNCERTAINTIES OF INFLATON FIELD WITH OBSERVED SPECTRUM OF INHOMOGENEITIES.

POINT OF THIS TALK: IT'S VERY INTERESTING, BUT NOT YET AN EXPLANATION.
WHAT IS NEEDED TO FULLY EXPLAIN?

THE STANDARD ACCOUNT: GRAVITY COUPLED TO SCALARFIELD

1) SPLIT g, ϕ INTO HOMOGENOUS PARTS g_0, ϕ_0 AND FLUCTUATIONS $\delta g, \delta\phi$
CONSIDER EINSTEIN EQUATIONS UP TO FIRST ORDER.
EX: GAUGE FIX, FORGET TENSOR MODES, SPATIALLY FLAT CASE

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)\,d\eta^2 + (1-2\Psi)\,\delta_{ij}\,dx^i dx^j\right]$$

2) WITH SUITABLE POTENTIAL FOR ϕ AND INITIAL CONDITIONS:

O ORDER EQUATIONS GIVE INFLATIONARY FRW SOLUTION FOR g_0, ϕ_0

I ORDER EQUATIONS: WAVE EQN FOR $\delta\phi$ ON BACKGROUND g_0 AND COUPLING TO δg. IN EXAMPLE:

$$\delta\phi'' + 3H\delta\phi' - a^{-2}\Delta\delta\phi = 0 \qquad \Delta\Psi \simeq 4\pi G a^2 \phi_0' \delta\phi'$$

3) QUANTIZE SCALAR FIELD, CHOOSE VACUUM.

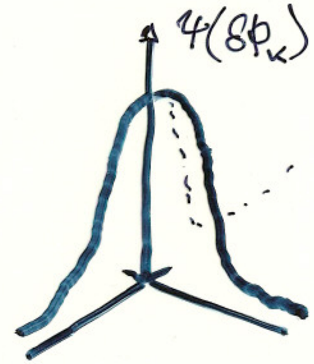
IN EXAMPLE, USE FOURIER TRANSFORM. FOR SIMPICITY ALSO: FIELD IN BOX.

$$\hat{\delta\phi}(t,x) = \sum_k c_k(t)\,\hat{a}_k\, e^{-ikx} + \bar{c}_k(t)\,\hat{a}_k^+ e^{ikx}$$

CHOICE OF VACUUM $\hat{=}$ SUITABLE CHOICE OF c_k

USUAL CHOICE: INSTANTANEOUS VACUUM AT EARLY TIMES.

i.E FOR MODE $\hat{\delta\phi}_k$ HO GROUND STATE (GAUSSIAN) AT EARLY TIME.



$$\Delta^2 \hat{\delta\phi}_k \Big|_{early} \simeq \frac{1}{2a^2k}$$

$$\Delta^2 \hat{\pi}_k \Big|_{early} \simeq \frac{\hbar a^2 k}{2}$$

4) EVOLUTION:

FOR $k > aH$ FRICTION TERM CAN BE NEGLECTED, ADIABATIC EVOLUTION

$$\delta\phi_k'' + 3H\phi_k' - \left(\frac{k}{a}\right)^2 \delta\phi_k = 0$$

FOR $k \simeq aH$ OVERDAMPED HO: ALMOST NO EVOLUTION ANYMORE.

AT THAT MOMENT („MODE LEAVES HORIZON")

$$\Delta^2 \hat{\phi}_k = \frac{H^2}{2k^3} \qquad \Delta^2 \hat{\pi}_k \simeq \frac{\hbar k^3}{2H^2}$$

5) THESE FLUCTUATIONS ARE IDENTIFIED WITH DENSITY FLUCTUATIONS.
EX: COMOVING ENERGY DENSITY FLUCTUATION

$$\delta\rho = \phi_0' \, \delta\phi' \quad \rightsquigarrow \quad |\delta\rho_k|^2 := a^{-6} \Delta^2 \hat{\pi}_k \simeq k^{-3}$$

↑ AT EXIT/REENTRY

THAT IS HARRISON-ZELDOVIC SPECTRUM.
(EQUIVALENTLY $\psi_k \sim k^{-\frac{3}{2}}$, $\delta\dot{\phi}_k \sim \sqrt{k}$ FOR EQUAL TIMES)

6) EVOLVE FURTHER (FULLY CLASSICAL), RESTRICT TO SURFACE OF LAST SCATTERING, COMPARE WITH NATURE.

THE ISSUES

DOES THAT EXPLAIN THE INHOMOGENEITIES? WHY IS STEP 5) RIGHT? NO EASY ANSWER:

- STATE AFTER INFLATION AS HOMOGENOUS AS BEFORE. (ONLY MORE „QUANTUM": SQUEEZED STATE)

- JUSTIFY BY $\delta G_{ab} = \langle \delta \hat{T}_{ab} \rangle$? NO: δT_{ab} LINEAR IN $\delta\phi$ AND $\langle \delta\hat{\phi} \rangle = 0 = \langle \delta\hat{\phi}_k \rangle$

- WMAP MEASURES CORRELATIONS, THAT MAY GIVE SQUARES:

$$\langle |\delta\hat{\phi}_k|^2 \rangle = \int e^{ikr} \int \langle \delta\hat{\phi}(x)\, \delta\hat{\phi}(x+r) \rangle \, dx \, dr$$

BUT ARE WE SURE WE MEASURE QUANTUM CORRELATIONS AND NOT $\delta\phi(x)$ SEPERATELY? AND WHY IDENTIFY EXPECTATION VALUE (≙ ENSEMBLE AVERAGE) WITH AVERAGES IN SKY?

- CAN WE APPLY STANDARD QM INTERPRETATION ANYWAY?

COLLAPSE OF WAVE FUNCTION ?!

NEED SOME PHYSICAL MECHANISM THAT CONVERTS QM UNCERTAINTIES INTO INHOMOGENEITIES. SOMETHING LIKE THIS SEEMS TO HAPPEN WHEN WE MEASURE OPERATORS NOT COMMUTING WITH SYMMETRY GENERATORS ...

SO CAN WE EXPLAIN INHOMOGENEITIES BY SOME „MEASUREMENT" / COLLAPSE ?

BUILD A PHENOMENOLOGICAL MODEL: ← A LOOONG SHOT !

- KEEP BACKGROUND CLASSICAL AS BEFORE
- QUANTIZE AND EVOLVE $\delta\phi$ AS BEFORE EXCEPT: SOMETIMES COLLAPSE HAPPENS
- COUPLE METRIC VIA $\delta G_{ab} = 8\pi G \langle \delta T_{ab} \rangle$
- FOR SIMPLICITY: FULLY SOLUBLE SITUATION (ETERNAL INFLATION)

MORE PRECISELY: FOR EACH MODE $\delta\phi_k$ ALLOW FOR MOMENT t_k^c IN WHICH TIME EVOLUTION OF STATE (SCHRÖDINGER PICTURE!) JUMPS

$$\langle \delta\hat{\phi}_k \rangle \Big|_{\text{AFTER}} = \sqrt{\Delta^2 \delta\hat{\phi}_k}\,\Big|_{\text{BEFORE}} \cdot x_k^{(1)}$$

$$\langle \hat{\pi}_k \rangle \Big|_{\text{AFTER}} = \sqrt{\Delta^2 \hat{\pi}_k}\,\Big|_{\text{BEFORE}} \cdot x_k^{(2)}$$

$x_k^{(1)}$, $x_k^{(2)}$: DISTRIBUTED ACCORDING TO GAUSSIAN W. SPREAD 1.

STICK YOUR FAVOURED MODEL HERE

(EACH SEPARATELY: STANDARD DESCR. OF MEASUREMENT, BUT NOT TOGETHER)

EVOLVE AND FIND WHEN MODE REENTERS

$$\langle \delta\dot{\phi}_k \rangle \sim \frac{1}{k^{3/2}} F\left(x_k, \frac{k}{a_k^c}\right)$$

HARRISON-ZELDOVICH $\Longleftrightarrow$ $\frac{k}{a_k^c} = \text{const.}$

SPECULATE EVEN MORE...

PENROSE MECHANISM ?

HAVE $\Delta\Psi = 4\pi G\, \dot{\phi}_0\, \delta\dot{\phi}$. INTERPRET Ψ LITTERALLY AS NEWTONIAN POTENTIAL, RHS LITTERALLY AS MASS DENSITY. THEN INTERACTION ENERGY BETWEEN QM ALTERNATIVES

$$E_I(t) = \int \Psi^{(1)}(x,t)\, \rho^{(2)}(x,t)\, a^3 d^3x$$

AND FOR ONE MODE =

$$E_I^k(t) = \frac{a^3}{L^3}\, \Psi_k^{(1)}\, \rho_k^{(2)} \simeq \frac{\pi}{4}\, \frac{a}{k}\, \hbar G\, (\dot{\phi}_0)^2$$

OPPOSITE SIDES OF GAUSSIAN

ASKING $E_I^k(t_k^c) = M_p$ GIVES $\frac{k}{a_k^c} \simeq \frac{\pi\hbar G \dot{\phi}_0^2}{M_p} \simeq \text{const}$.

SUMMARY

HOPE I HAVE CONVINCED YOU THAT

- THERE IS SOMETHING REMARKABLE GOING ON BETWEEN QUANTUM AND CLASSICAL FLUCTUATIONS
- WE DON'T YET UNDERSTAND THE FULL PICTURE
- MAYBE WE NEED SOME NEW PHYSICS
- THERE IS AMPLE ROOM FOR FUN SPECULATIONS