ON THE PERTURBATIVE EXPANSION OF YANG-MILLS THEORY AROUND BF THEORY

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QFT 2-point functions

plus QG corrections

Long term project:

$$W_{\mu\nu}(x,y) = \int DA\, Dg\; A_\mu(x) A_\nu(y)\, e^{\frac{i}{\hbar}\left(S_{GR}[g] + S_{YMO}[g,A]\right)}$$

$$= W^{(0)}_{\mu\nu}(x,y) + g_0 \ell_{Pe}\, W^{(1)}_{\mu\nu}(x,y) + \dots$$

$$\downarrow$$

$$\langle 0 | A_\mu(x) A_\nu(y) | 0 \rangle = FT\left(\frac{1}{p^2}\right)$$

GRAVITON FROM SPINFOAMS

Modesto-Rovelli: gr-qc/0502036

Rovelli: gr-qc/0508129

$$W_{\mu\nu\rho\sigma}(x,y;q) = \sum_s \langle s | h_{\mu\nu}(x) h_{\rho\sigma}(y) | s \rangle \, \Psi_q[s] \, W_0[s]$$

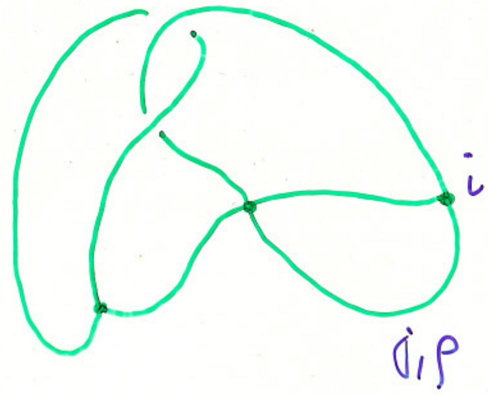
(i) field insertions

(ii) "vacuum state" peaking the spin networks around a classical background q

(iii) (Linearised) propagation kernel

PHOTON FROM SPINFOAMS

spin network for GR and YM



$$|s\rangle = |\gamma, j_e, \rho_e, i_n\rangle$$

$$W_{\mu\nu}(x,y) = \sum_s \langle s| U_\mu(x) U_\nu(y) |s\rangle \, \Psi_q[s] \, \Psi_0[s] \, W_0[s]$$

(i) field insertions

(ii) "vacuum grav. state", peaking the grav. labels of $|s\rangle$ around q

(iii) "vacuum YM state" (simple on the continuum, very complicated on a lattice)

(iv) (Linearised) propagation kernel

to be read from a spinfoam model

YM AS A MODIFIED BF THEORY

- Second order action

$$S = \frac{1}{2g_0^2} \int F \wedge * F$$

- First order action

$$S = \int B \wedge F + \frac{g_0^2}{2} \int B \wedge * B$$

$$\Big\downarrow$$

Halpern PRD 16 ('77)
Schaden et. al. NPB 339 ('90)
Cattaneo et al. CMP 197 ('98)
(BFYM theory)

define a spin foam model via a perturbative expansion in $g_0^2 G$

(Freidel, Rovelli, Speziale, work in progress)

$$W[s] = \sum_{n=0}^{\infty} \frac{g_0^n}{n!} W^{(n)}[s]$$

FIRST ORDER ACTION FOR YM

Does it make sense to expand a non-trivial qft around a topological theory?

Consider <u>flat</u> spacetime

$$\Gamma^{YM}_{\mu\nu}(x,y) = \int DA\, A_\mu(x) A_\nu(y)\, e^{-\int (dA + g_0 A\wedge A)^2 - S_{gf}(A)}$$

$$\Gamma^{BFYM}_{\mu\nu}(x,y) = \int DA\, DB\, A_\mu(x) A_\nu(y)\, e^{-i\int B\wedge F + \frac{g_0^2}{2}\int B\wedge * B - S_{gf}(B,A)}$$

from the formal equivalence

$$\Gamma^{YM}_{\mu\nu} = \frac{1}{g_0^2}\Gamma^{BFYM}$$

$$\Rightarrow \Gamma^{YM(0)} + g_0^2\,\Gamma^{YM(1)} + \dots = \frac{1}{g_0^2}\left[\Gamma^{BFYM(0)} + g_0^2\,\Gamma^{BFYM(1)} + \dots\right]$$

$$= \int \mathcal{D}A \, A_\mu(x) A_\nu(y) \, e^{-\int (dA)^2 - S_{gf}(A)}$$

$$+ \sum_{n=1}^{\infty} \frac{g^n}{n!} \Gamma^{(n)}_{\mu\nu}(x,y)$$

$\left\{ \begin{array}{l} \text{abelian} \\ \text{metric} \end{array} \right.$

$$= \int \mathcal{D}A \mathcal{D}B \, A_\mu(x) A_\nu(y) \, e^{-i\int B\wedge F - S_{gf}(B,A)}$$

$$+ \sum_{n=1}^{\infty} \frac{g^n}{n!} \Gamma^{BFYM\,(n)}_{\mu\nu}(x,y)$$

$\left\{ \begin{array}{l} \text{non-abelian} \\ \text{topological} \end{array} \right.$

COMPUTATION OF THE ABELIAN PROPAGATOR

$$\Gamma^{BFYM\,(1)}_{\mu\nu}(x,y) = -\frac{1}{2}\frac{\delta^2}{\delta j^\mu(x)\,\delta j^\nu(y)} Z[j]\Big|_{j=0}$$

$$Z[j] = \int \mathcal{D}A\mathcal{D}B \int B\wedge * B \; e^{-i\int B\wedge dA - S_{gf}(A) + i\int j\cdot A}$$

$$= \int \mathcal{D}B \int B\wedge * B \;\delta(*dB + j_T)$$

Fourier space: $\varepsilon^{\mu\nu\rho\sigma} B_{\rho\sigma} = \frac{2}{p^2}\, p^{[\mu} j_T^{\nu]}(p)$

$$= \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\,\delta_{\mu\nu}\, j_T^\mu(p)\, j_T^\nu(-p)$$

$$j_T{}^\mu = D^\mu{}_\nu j^\nu, \quad D^\mu{}_\nu = \delta^\mu{}_\nu - \frac{p^\mu p_\nu}{p^2}$$

$$\Rightarrow \int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x-y)}}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

COMPUTATION OF THE FREE PROPAGATOR

$$\Gamma^{YM(0)} \overset{?}{=} \Gamma^{BFYM(1)}$$

$$= \frac{1}{2} \int DADB\, A_\mu(x) A_\nu(y) \int B \wedge * B \, e^{-i\int B\wedge F - S_{gf}(B,A)} =$$

$$= \frac{1}{8} \frac{\delta^2}{\delta j^\mu(x) \delta j^\nu(y)} \int dz \frac{\delta}{\delta \eta^{\rho\sigma}(z)} \frac{\delta}{\delta \eta_{\rho\sigma}(z)} Z[j,\eta] \Big|_{j=\eta=0}$$

$$Z[j,\eta] = \int DADB\, e^{-i\int B\wedge F - S_{gf}(B,A) + i\int j\cdot A + i\int \eta\cdot B}$$

$$= \int DA\, \delta(*F - \eta)\, e^{-S_{gf}(A)}\, e^{i\int j\cdot A}$$

$$= e^{i\int j\cdot A_\eta}$$

$$= \int dz \frac{\delta A_{\eta\,\mu}(x)}{\delta \eta^{\rho\sigma}(z)} \frac{\delta A_{\eta\,\nu}(y)}{\delta \eta_{\rho\sigma}(z)} \Big|_{\eta=0}$$

$$\begin{cases} F_{\mu\nu}(A_\eta) = \varepsilon_{\mu\nu\rho\sigma}\,\eta^{\rho\sigma} \\[2em] A_{\eta\,\mu}(x) = \int dx^1\, G^{(1)}_{\mu\nu\rho}(x,x^1)\,\eta^{\nu\rho}(x^1) + \end{cases}$$

$$+ \int dx^1 dx^2\, G^{(2)}_{\mu\,\nu\rho\sigma\tau}(x,x^1,x^2)\,\eta^{\nu\rho}(x^1)\,\eta^{\sigma\tau}(x^2)$$

$$+ \dots$$

Leading order:

$$\partial_{[\mu}\, G^{(1)}_{\nu]\rho\sigma}\,\eta^{\rho\sigma} = \varepsilon_{\mu\nu\rho\sigma}\,\eta^{\rho\sigma}$$

$$\left\{\begin{array}{l} \text{Lorentz gauge} \\ \text{Fourier space} \end{array}\right. \longrightarrow G^{(1)}_{\nu\rho\sigma}(p)\,\eta^{\rho\sigma}(p) = \frac{p^\mu}{p^2}\,\varepsilon_{\mu\nu\rho\sigma}\,\eta^{\rho\sigma}$$

and finally

$$\Gamma^{BFYM\,(1)}_{\mu\nu}(x,y) = \int dz\, G^{(1)}_{\mu\rho\sigma}(x,z)\, G^{(1)}_{\nu\rho\sigma}(y,z) =$$

$$= \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip(x-y)}}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Dependence on the non-abelian structure at 2nd order

CONCLUSIONS

- The perturbative expansion for $g_0 \ll 1$ of the action
$$\int B \wedge F + \frac{g_0^2}{2} \int B \wedge * B$$
is equivalent to the conventional
$$\int (dA + g_0 A \wedge A)^2$$

- The zeroth order does not contribute
- The leading term is the first order
$$\int \mathcal{D}A \mathcal{D}B \, A_\mu(x) A_\nu(y) \int B \wedge * B \; e^{-i \int B \wedge F}$$
- The non-abelian structure enters at NLO,
$$\int \mathcal{D}A \mathcal{D}B \, A_\mu(x) A_\nu(y) \left[\int B \wedge * B \right]^2 e^{-i \int B \wedge F}$$

$\Rightarrow$ Viability of this approach in spin foams.