# Dynamics of the Bouncing Cosmological Models

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## Overview

1. The bouncing cosmological models as a dynamical system
2. Structural instability of bouncing cosmology
3. How do bouncing universes fit to supernovae data
4. The degeneration problem in observational cosmology
5. Information criteria — a tool to descriminate between cosmological models
6. Conclusion — the ΛCDM model or the bouncing model

talk base on the paper M.Sz. et al
" Can the initial singularity be detected by cosmological tests?
Phys.Rev D72, 0 63504 (2005)

# Testing the cosmological models with acceleration

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**Abstract**

The Unified approach to investigation of dynamics of the cosmological models with dark energy is presented. We have proposal to use the potential function in the hamiltonian formulation of FRW dynamics as a probe of different dark energy models. The potential function can be also reconstructed from SNIa data. We discuss how many of effential parameters of the models are statisticaly significant in means of the Akaike and Bayesian informative criteria. We show that the hierarchy of cosmological models can be established in the ensemble of dark energy models by informative criteria. We argue that SNIa data support that the number of essential parameters (or the dimension of phase space) is two, i.e. $(H_0, \Omega_{m,0})$.

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# V-reconstruction method—a direct and inverse problem in the quintessential cosmology

### Particle-like description of FRW cosmology

We start from the Friedmann-Robertson-Walker (FRW) models filled with perfect fluid with the equation of state $p = w(a)\rho$, where $p$ and $\rho$ are its pressure and energy density. We assume that the coefficient $w$ in the equation of state is parameterized by scale factor $a$. The basic dynamical equation in this problem (called as a direct dynamical problem) constitute two equations

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \tag{1}$$

$$\dot{\rho} = -3H(\rho + p) \tag{2}$$

The first equation is a consequence of the Rauchaudhuri equation while the second one the conservation condition.

Then we can reduce equation (1) to the form analogous to the Newtonian equation of motion for a particle moving in one-dimensional potential $V(a)$

$$\ddot{a} = -\frac{\partial V}{\partial a} \tag{3}$$

where

$$V(a) = \frac{1}{6}\int(\rho + 3p)a da \tag{4}$$

is the potential function and the scale factor $a$ plays the role of a positional variable in the configuration space $\{a : a \geq 0\}$.

The integration of (4) by parts gives

$$V(a) = \frac{1}{12}\left[(\rho + 3p)a^2 - \int a^2 d(\rho + 3p)\right] = \frac{1}{12}\left[(\rho + 3p)a^2 - 3(\rho + p)a^2\right]. \tag{5}$$

Two appraches to solving the problem of acc. of the Universe

1) Universe is accelerating due to presence of dark energy of unknown form violating the SEC

2) Universe is accelerating due to the modified FRW equations, for example by quantum corrections, exotic physics

V-reconstruction method—a direct and inverse problem in the quintessential cosmology

## Particle-like description of FRW cosmology

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The last term in (5) can be obtained in the exact form if we rewrite (2) to the equivalent form

$$a^3 dp = d[(\rho + p)a^3] \tag{6}$$

Finally, we obtain the potential in terms of (effective) energy density in the form (modulo an arbitrary constant)

$$V(a) = -\frac{\rho}{6}a^2 \tag{7}$$

and the function of energy is preserved

$$\mathcal{E} = \frac{\dot{a}^2}{2} + V(a) = -\frac{k}{2} \tag{8}$$

where $k$ is a constant of curvature. Note that the function $V(a)$ plays role of the potential for a fictitious particle of unit mass which mimics the evolution of the universe. The Hamilton function is

$$\mathcal{H}(p_a, a) = \frac{1}{2}p_a^2 + V(a). \tag{9}$$

The motion of the system is restricted to the level $\mathcal{H} = 0$, if we include the curvature contribution in $\rho_{\text{eff}}$.

Therefore, if we consider the FRW dynamics for the universe filled with fluid with the general form of the equation of state factor $w(a(z)) = p/\rho$ then dynamics can be reduced to the two-dimensional dynamical system

$$\dot{a} = x \tag{10}$$

$$\dot{x} = -\frac{\partial V}{\partial a} \tag{11}$$

where $V(a)$ is given by (7) and Friedmann first integral (8) is the first integral of (10)-(11). The potential $V(a)$ identifies the model under consideration.

System (10)-(11) has critical points at $x_0 = 0$ and $a = a_0$: $\frac{\partial V}{\partial a}|_{a=a_0}$ and $V(a_0) = 0$. It represents the static Einstein universe. The domain admissible for motion is $\{a: V(a) \leq 0\}$.

Note that the localization of the critical points as well as its character is determined from the geometry of the potential function only. The linearization of system (10)-(11) in the neighbourhood of the critical point gives

$$\dot{x} = -\left.\frac{\partial^2 V}{\partial a^2}\right|_{a_0} x \qquad \text{or} \qquad \ddot{a} = -\left.\frac{\partial^2 V}{\partial a^2}\right|_{a_0} (a - a_0). \tag{12}$$

Because the eigenvalues of the linearization matrix $\lambda_{1,2} = \pm\sqrt{-\left.\frac{\partial^2 V}{\partial a^2}\right|_{a_0}}$ can be real of opposite signs or purely imaginary, then a saddle or a centre are only admissible, respectively.

From the reconstruction of SNIa data we obtain that V(a) is a upper convex function. Therefore, the system is structurally stable.

### Reconstruction of the potential $V(a)$ from distant supernovae

Further in this section we use the flat model ($k = 0$) since the evidence for this case is very strong in the light of WMAP data. For our aims it would be useful to rearrange the Friemann first integral to the form giving the Hubble function

$$H(z) = \sqrt{-2V(a(z))(1+z)^2} = \sqrt{\frac{\rho_{\text{eff}}}{3}} \tag{13}$$

where we express in terms of redshift $(1+z) = a^{-1}$.

To confront the theoretically assumed potential $V$ with SNIa data we calculate the luminosity distance in the standard way

$$d_L(z) = (1+z)\int^{z} \frac{dz'}{H(z')}. \tag{14}$$

Because we are going to reconstruct $V(a)$ and compare its form with the theoretically suggested one, it would be useful to take the inverse of formula (14)

$$H(z) = \left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}. \tag{15}$$

Hence

$$V(a(z)) = -\frac{1}{2}(1+z)^2\left[\frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right]^{-2}. \tag{16}$$

In the search for different dark energy models the method of reconstruction of $w(z)$ is very popular. There are many problems with the realization of this idea because of the smearing effect

Our key idea is to use the potential $V(a)$ (or effective energy density) instead of $w(z)$ for the characterization of dark energy models. We presented this procedure and it was applied to the case of the model with Chaplygin gas and the Cardassian model

The main advantage of using the $V$-method is the enclosing of all information about the dynamics in the geometry of the potential function. Hence, the energy density $\rho_{\text{eff}}(z)$ can be simply obtained from $V(z)$. The effectiveness of the $V$-method will be demonstrated in the next section.

## 3. Potential function from SNIa data

In the statistical analysis the Knop et al.'s sample of distant supernovae type Ia data was used . Figure 1 shows the potential function reconstructed directly from the data with denoted regions of levels of uncertainties $1\sigma$ and $2\sigma$. Given the potential we can obtain immediately the phase portrait (Fig. 2).

Let us summarize the theoretical and practical advantages of using $V$ method. First, all information about the dynamics is included in the geometry of a single potential calculations like the tunnelling amplitude from the Gamov formula can be done.

Second, from the potential function is always possible to extract the information about the matter content from the following formula $\rho(t) = -6Va^{-2}$, $p(t) = 2V(1+I_V(a))a^{-2}$, $w(a(z)) = -(1+I_V(a(z)))/3$, where $I_V(a) = d\ln V/d\ln a$ is the elasticity coefficient of the potential function with respect to the scale factor.

Third, many investigations allows to find the constraint on the coefficient $w_X$ of dark energy component $X$ from the distant supernovae data . Because of the smearing effect coming from the presence of the double integrals relating $w(z)$ to the luminosity distance $d_L$. It is extremely hard to constraint the $w_X$. We propose the potential function instead of $w_X$ for probing the dark energy because in this case $w_X$ is expressed by only a single integral. Therefore it is easier to obtain this kind of constraints. It is the main practical advantage.

Fourth, in the reconstruction $V(z)$ we fit the polynomial function, however, if one can demonstrate that $V$ method is not sensitive on the choice of parameterization of $w(z)$ . In all cases we obtain the diagram of function $V(a)$ which is upper convex.

Fifth, the maximum of the potential function represents the important moment during the evolution of the Universe when the transition from the decelerating to accelerating phase takes place. It is interesting that the value of redshift at this moment can be estimated from the SNIa data with good accuracy

Let us consider the potential function of class $C^2$. Apart from the reconstructed potential from observational data we can obtain such a potential for theoretical models of dark energy. In the class of potential one can introduce the distance $d(X,Y) = |X - Y|$ with the help of norm defined as

$$||X|| = \max\left\{\sup_{a\in I}|X(a)|, \sup_{a\in I}|\partial X(a)/\partial a|, \sup_{a\in I}|\partial^2 X(a)/\partial a^2|\right\}$$

where $I$ is the assumed interval of redshift.

The space of all dark energy models equipped with such a metric is the Banach space .

Let $\tilde{V}$ be the best fit potential reconstructed from the SNIa data. Using this metric we can measure the distance of every potential from the theory to the potential $\tilde{V}$. The space of all models with dark energy is the Banach space with well defined distance between the models. This space has natural topology defined with the help of the metric.

Basic problem is

Whether LQC can explain the present acceleration of the Universe
We known (Bojowald 2003, 2004) that LQC has exit on accel. epoch but from SNIa obs. we estimate that Univ. start to accel. at $z_T \approx 0.5$

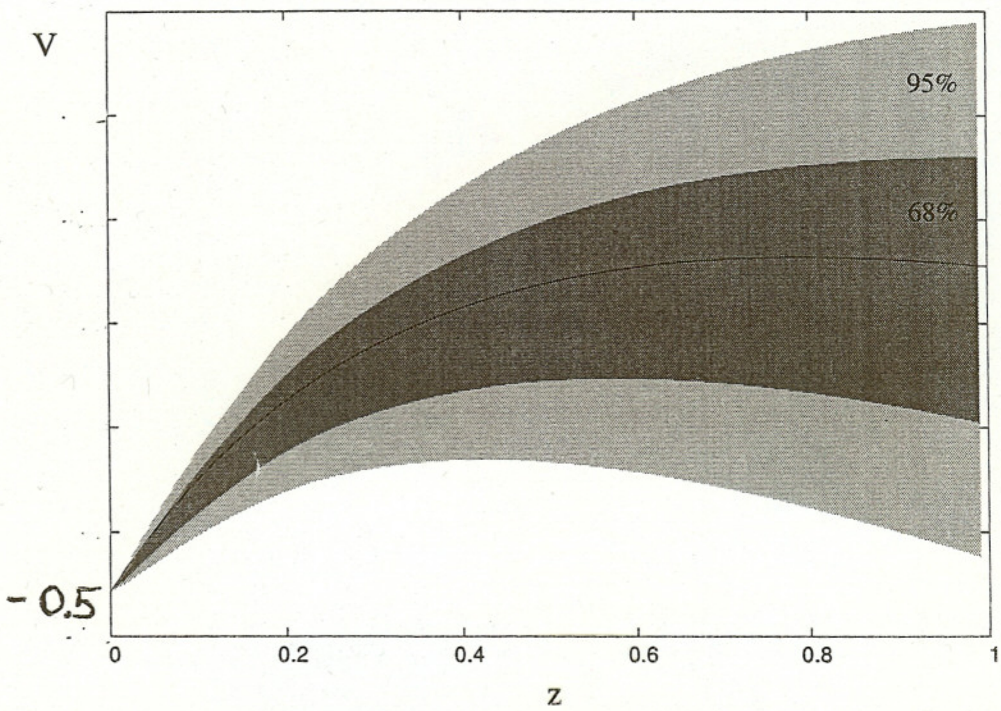



Fig. 1. The potential function for the reconstructed best fit model is given by a solid line. Around it the confidence regions $1\sigma$ and $2\sigma$ are given.

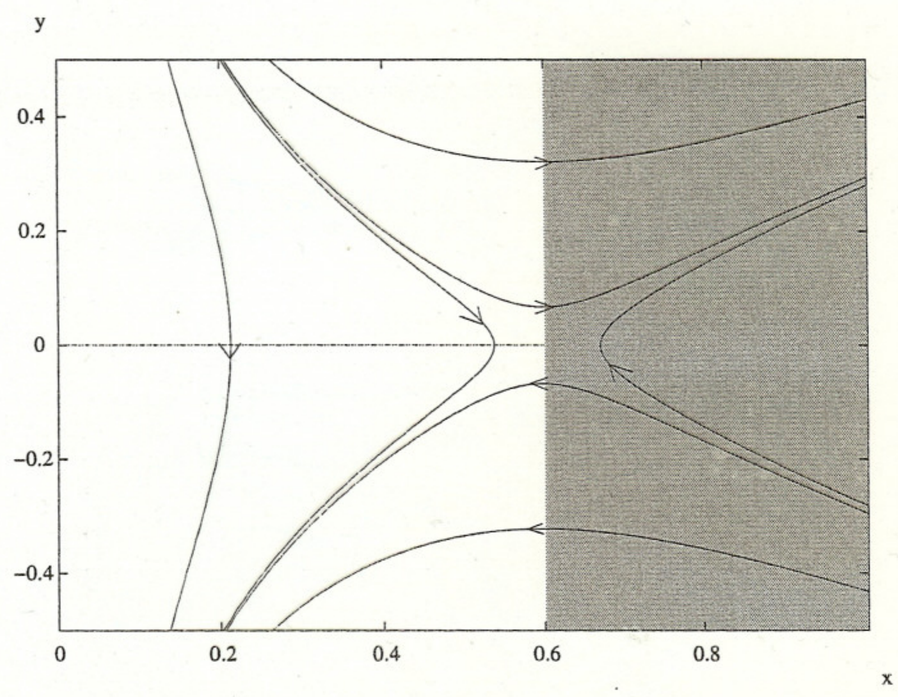



Fig. 2. The phase portrait from the reconstructed potential. The saddle type of critical point represent the Einstein static universe. The shaded region represents the accelerating domain.

function. It is obvious for the whole class of FRW models filled with fluid satisfying the equation of state in the form $p = w(a(z))\rho$. The fact that its dynamics can be reduced to motion of the single particle in the one-dimensional potential can be a starting point of the further analysis exploring this analogy. Moreover, some

## Example $\Lambda$CDM model

$$\rho_{eff} = \Lambda + \rho_{m,0}\, a^{-3} \quad , \rho_{m,0} = \rho(a = a_0 = 1)$$

$$p_{eff} = -\Lambda + 0$$

$$V = -\frac{\rho_{eff} a^2}{6} = -\frac{\Lambda a^2}{6} - \frac{\rho_{m,0} a^{-1}}{6} + \frac{k}{2}$$

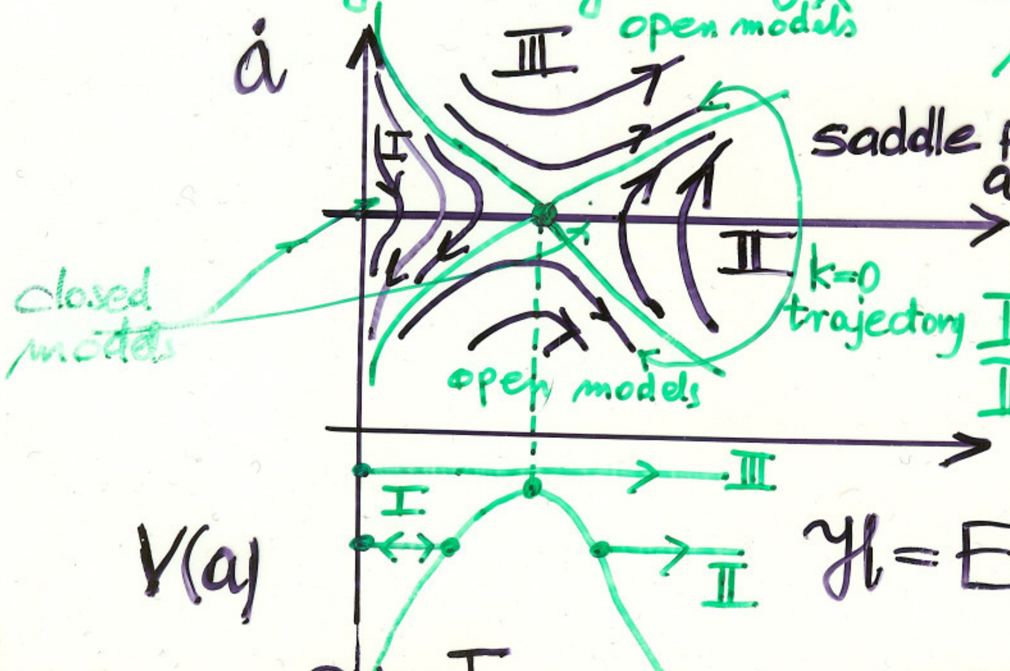
$$= H_0^2 \left\{ -\frac{1}{2}\Omega_{\Lambda,0} x^2 - \frac{1}{2}\Omega_{m,0} x^{-1} - \frac{1}{2}\Omega_{k,0} \right\}$$

$$1 + z = \frac{a_0}{a} \equiv x^{-1}, \quad \sum_{i=(\Lambda,m,k)} \Omega_{i,0} = 1$$

$$\dot{x}^2 + V(x) = \frac{1}{2}\Omega_{k,0}, \quad \dot{} \equiv \frac{d}{d\tau} : H_0 dt = d\tau$$

$$\dot{x} = y, \quad \dot{y} = -\frac{\partial V}{\partial x}$$

Newtonian dyn system

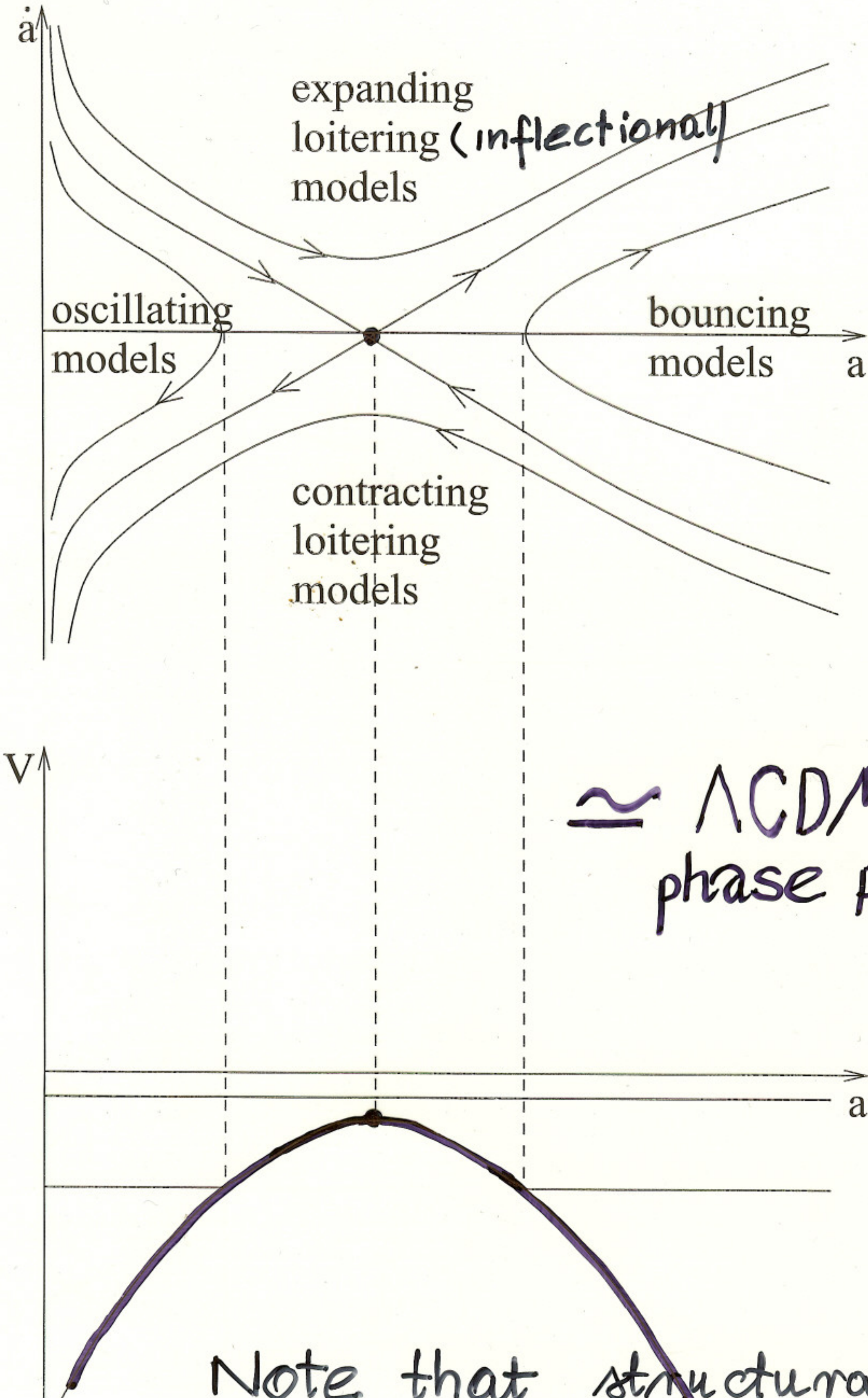



I oscillating models
II bouncing models
III loitering model

$\mathcal{H} = E = const$

$f = \left[ y, -\frac{\partial V}{\partial x} \right]^T$ is structurally stable – for any vector $g$ near $f$, the vectors field $f$ & $g$ are top. equivalent

| case | potential of the model | model parameters |
|---|---|---|
| (1) model with Chaplygin gas $p=-\frac{A}{\varrho^{\alpha}}$, $A>0$ | $V(x)=\left(-\frac{1}{2}\right)\left\{\Omega_{m,0}x^{-1}+\Omega_{Chapl,0}\left(A_s+\frac{1-A_s}{x^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}\right.$ | $\sum_i \Omega_{i,0}=1$ $(\Omega_{m,0};\Omega_{ch,0},A_s,\alpha)$ |
| (2) Cardassian models (or two noninteracting fluids) | $\varrho_{eff}=\varrho+3B\varrho^{n}$, $\varrho=\varrho_{m,0}a^{-3(1+w)}$ $V(x)=\left(-\frac{1}{2}\right)\left\{\Omega_{w,0}x^{-1-3w}+\Omega_{card,0}x^{-3(1+w)n+2}+\Omega_{k,0}\right\}$ | $(\Omega_{card};\Omega_{w,0},\Omega_{k,0})$ $\sum_{(w,card,k)}\Omega_{i,0}$ |
| (3) models with dyn E.Q.S. linearized around $z=0$ or $a=1$ $w_x(z)=w_0+w_1z$ $w_x(a)=w_0+w_1(1-a)$ | $\varrho_{eff}=\varrho_{m,0}(1+z)^3+\varrho_{x,0}e^{3w_1z}(1+z)^{3(w_0-w_1+1)}$ $V(z)=\left(-\frac{1}{2}\right)\left\{\Omega_{m,0}(1+z)+\Omega_{x,0}e^{3w_1z}(1+z)^{3(w_0-w_1)+1}\right\}$ | $(\Omega_{m,0};\Omega_{x,0},w_0,w_1)$ |
| (4) bouncing models $3H^2=\frac{A}{x^m}-\frac{B}{x^n}+\Lambda$ $n>m$; $A,B>0$ | $\varrho_{eff}=Ax^{-m}-Bx^{-n}+\Lambda$ $V(x)=\left(-\frac{1}{2}\right)\left\{\Omega_{m,0}x^{-m+2}+\Omega_{m,0}x^{-n+2}+\Omega_{\Lambda,0}x^{2}\right\}$ | $(\Omega_{m,0};\Omega_{m,0},m,n)$ |




$\simeq$ ΛCDM
phase portrait

Note that structurally
stable scenarions of evolut.
is decell.$^{0\text{-}1}$ phase – accel. phase
$z_T \simeq 0.5$ from SNIa obs.
not acc – decc – acc like suggest LQC

| case | name of model | $H(z)$ | free parameters | $d$ |
|---|---|---|---|---|
| 0 | Einstein-de Sitter | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2}$ | $H_0, \Omega_{\mathrm{m},0}$ | 2 |
| 1 | ΛCDM | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_\Lambda}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_\Lambda$ | 3 |
| 2 | TDCDM | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{T,0}(1+z)}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{T,0}$ | 3 |
| 3a | PhCDM, $w=-\frac{4}{3}$ | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{Ph,0}(1+z)^{3(1+w)}}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{Ph,0}$ | 3 |
| 3b | PhCDM, $w$ – fitted | | $H_0, \Omega_{\mathrm{m},0}, \Omega_{Ph,0}, w$ | 4 |
| 4a | BΛCDM, $n=6$ | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2-\Omega_{n,0}(1+z)^n+\Omega_\Lambda}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_\Lambda$ | 4 |
| 4b | BΛCDM, $n$ – fitted | | $H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_\Lambda, n$ | 5 |
| 5a | BPhCDM, $n=6$ | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2-\Omega_{n,0}(1+z)^n+\Omega_{Ph,0}(1+z)^{-1}}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_{Ph,0}$ | 4 |
| 5b | BPhCDM, $n$ – fitted | | $H_0, \Omega_{\mathrm{m},0}, \Omega_{n,0}, \Omega_{Ph,0}, n$ | 5 |
| 6 | BrΛCDM, $n=6$ | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{BRA,0}(1+z)^n+\Omega_\Lambda}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{BRA,0}, \Omega_\Lambda$ | 4 |
| 7a | DEQS, $w_0 = -1$, $p_X=(w_0+w_1 z)\rho_X$ | $H = H_0\sqrt{\Omega_{\mathrm{m},0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{X,0}(1+z)^{3(w_0-w_1+1)}e^{3w_1 z}}$ | $H_0, \Omega_{\mathrm{m},0}, \Omega_{X,0}, w_1$ | 4 |
| 7b | DEQS, $w_0$ = fitted, $p_X=(w_0+w_1 z)\rho_X$ | | $H_0, \Omega_{\mathrm{m},0}, \Omega_{X,0}, w_0, w_1$ | 5 |

Table 1. The Hubble function versus redshift for the seven evolutional scenarios of the FRW models with dark energy.

# Which cosmological model with dark energy — phantom or $\Lambda$CDM?

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**Abstract**

In cosmology many dramatically different scenarios with the past (big bang versus bounce) and in the future (de Sitter versus big rip) singularities are compatible with the present day observations. This difficulty is called the degeneracy problem. We use the Akaike and Bayesian information criteria of model selection to overcome this degeneracy and to determine a model with such a set of parameters which gives the most preferred fit to the SNIa data. We consider seven representative scenarios, namely: the CDM models with the cosmological constant, with topological defect, with phantom field, with bounce, with bouncing phantom field, with brane and model with the linear dynamical equation of state parameter. Applying the model selection information criteria we show that AIC indicates the flat phantom model while BIC indicates both flat phantom and flat $\Lambda$CDM models. Finally we conclude that the number of essential parameters chosen by dark energy models which are compared with SNIa data is two.

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**Df.** Ensemble of FRW models with dark energy is defined as a space of all 2D dyn. systems of Newtonian type:

$\dot{x} = y$, $\dot{y} = -\frac{\partial V}{\partial x}$, $y^2/2 + V(x) = E = \frac{1}{2}\Omega_{k,0}$

, where potential of the system $V(x)$ is taken from dark energy models

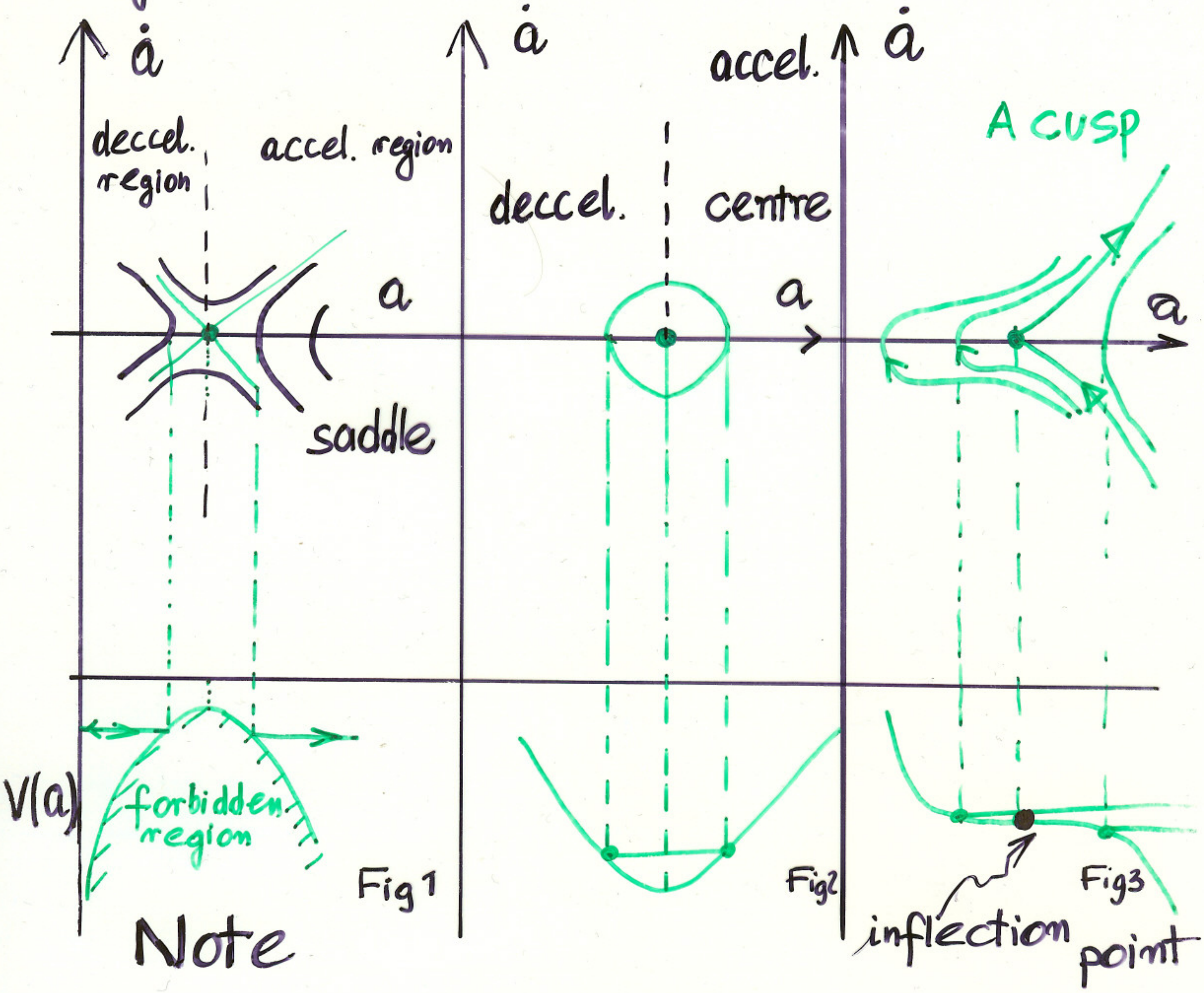
- The critical points are representing static solution: $y_0 = 0$, $x = x_0$, $V(x_0) = E$

•• If $(x_0, 0)$ is a strict <u>local maximum</u> of $V(x)$, it is a saddle point

••• If $(x_0, 0)$ is a strict <u>local minimum</u> of the $V(x)$, it is a center

•••• If $(x_0, 0)$ is a horizontal <u>inflection points</u> of the $V(x)$ it is a cusp

All these cases are illustrated on the Fig 1, 2, 3.




Note

- Because $\frac{\partial V}{\partial a} = \frac{1}{6}(\varrho_{eff} + 3p_{eff}) = (-)\ddot{a}$ Universe is accelerating in the domain in which potential is decreasing function of scale factor $a$.

# Generic and Nongeneric models in the Ensemble of Dark En. Models

Let $f$ is $C^1$ vector field defined on open subset $E$ of ensemble a $\mathcal{E}$, then $C^1$ norm of $f$ $\left( f = [y, -\frac{\partial V}{\partial x}]^T \right)$ can be introduced in standard way (so called Sobolev norm)

$$\|f\|_1 = \sup_{x \in E} |f(x)| + \sup_{x \in E} \|Df(x)\|$$

Perko p. 312

or

$$\|f\| = \max \left\{ \sup_{x \in E} (|y|), \sup_{(x,y) \in E} \left(\left|-\frac{\partial V}{\partial x}\right|\right), \sup_{(x,y) \in E} \left(\left|-\frac{\partial^2 V}{\partial x^2}\right|\right), 1 \right\}$$

Smale 1980 p. 60

The vector field with the metric $d(X, Y) = \|X - Y\|$ is Banach space.

$$\mathcal{E} \equiv (\{f\}, d)$$

$\{\ldots\}$ is representing abstraction class of the eqiv. rel.

This model can be rulled out by SNIa data (Riess Gold sample) on $4\sigma$ confidence level

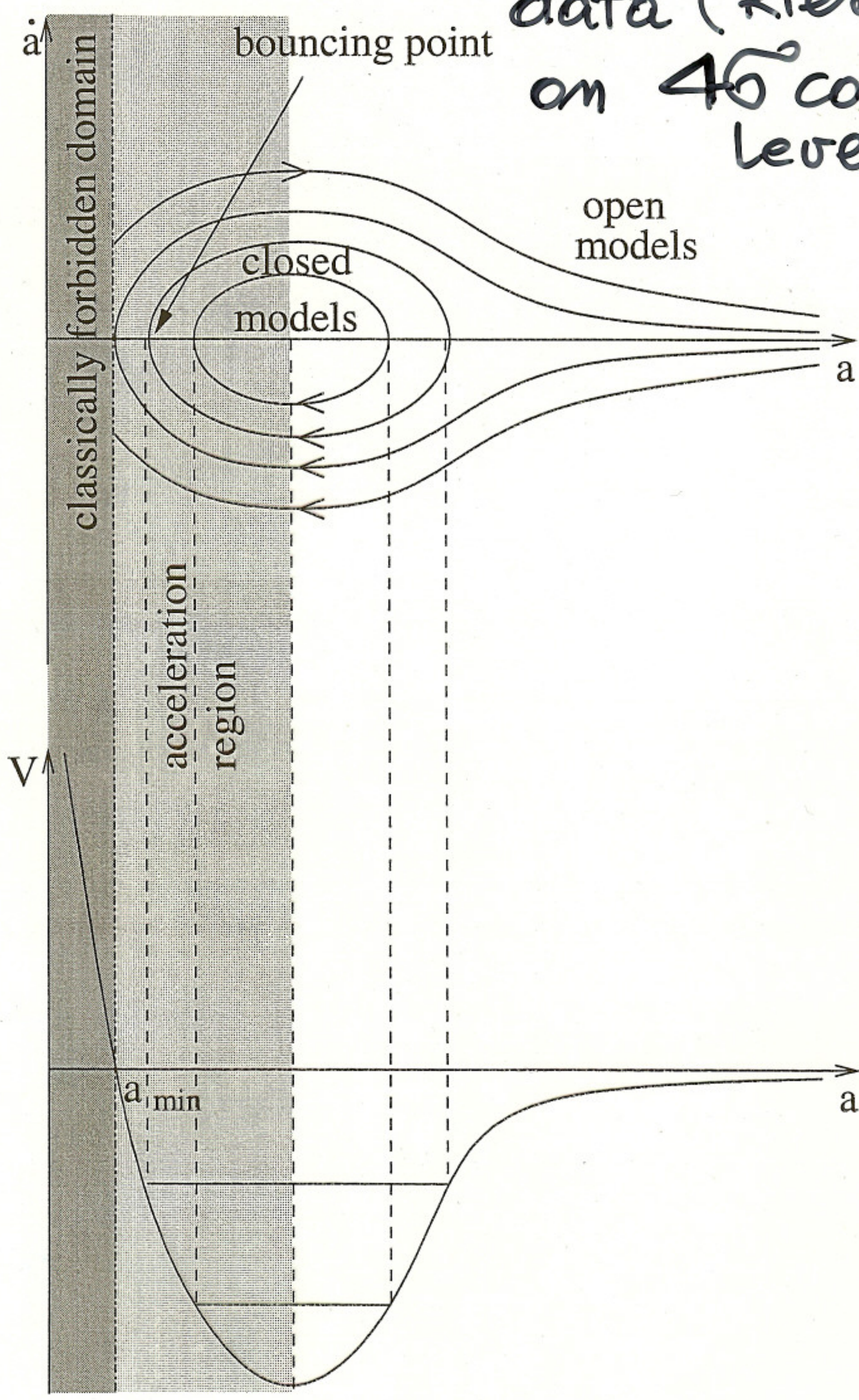



FIG. 1: The phase portrait and the diagram of the potential function for BM model (all case from Table I). The minimum of the potential function corresponds to a centre on the phase plane. The acceleration region is located on the right from the $a_{\min}$

$$H^2 = \Omega_{m,0}(1+z)^m - \Omega_{n,0}(1+z)^n$$



$\Omega_{m,0}, \Omega_{n,0} > 0$ ; $n > m$, a is expressed in the unit of its present value $a_0 = 1$

$1 + z = a_0/a$

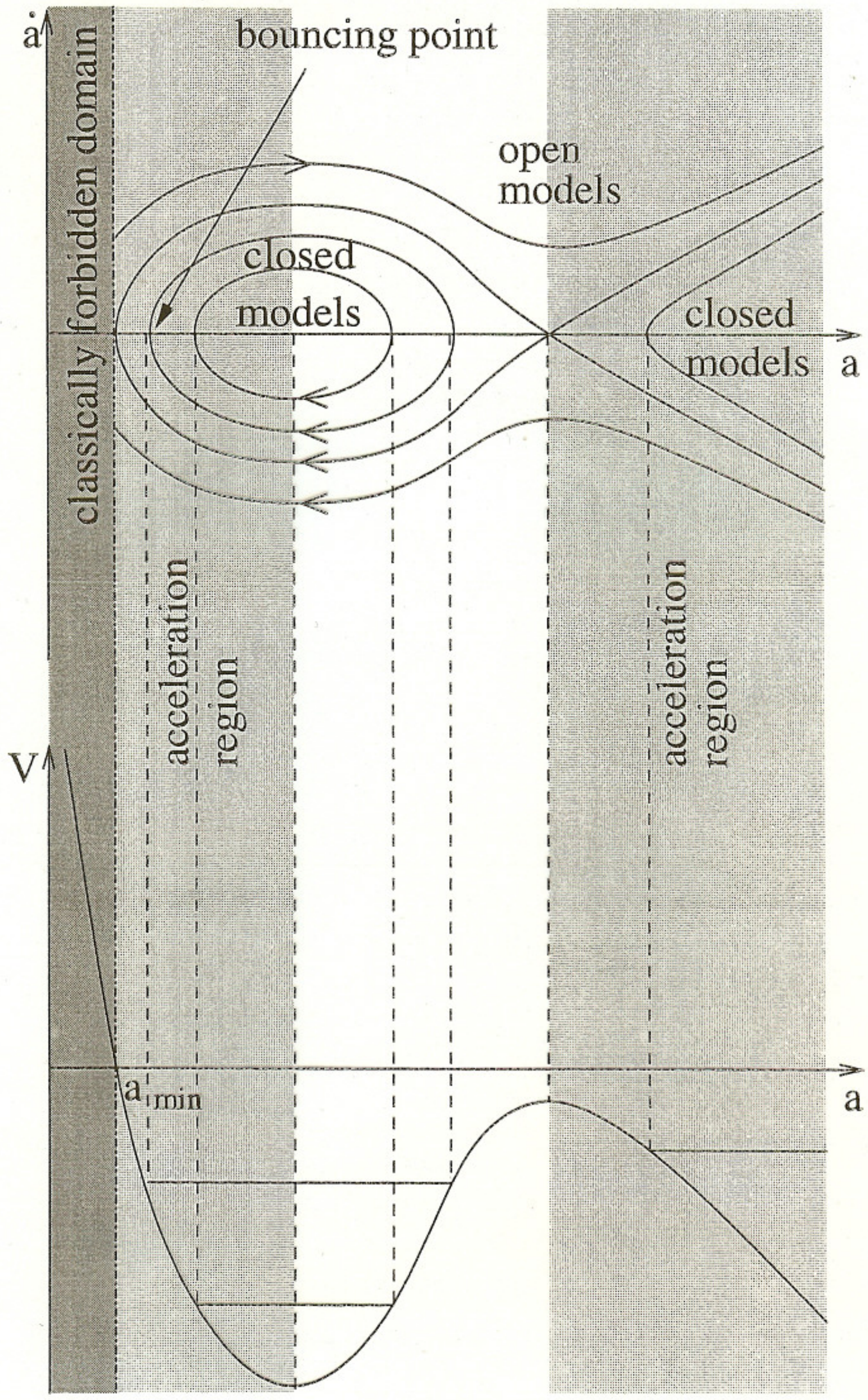



FIG. 2: The phase portrait and the diagram of the potential function for ΛBCDM model. The minimum (maximum) of the potential function corresponds to a centre (saddle) on the phase plane. The system is structurally unstable because of the presence of nonhyperbolic critical point (a centre).

While there is no counter part to Peixoto Theorem in higher dim. it can be easy to test whether such planar polynomial system has structurally stable phase portrait

The behaviour of trajectories at infinity is performed in tools of Poincare sphere $S^2$ construction

: we project trajectories from the centre of the unit sphere

$$S^2 = \{(X,Y,Z) \in R^3 : X^2+Y^2+Z^2=1\}$$

onto the $(x,y)$ plane tangent to $S^2$ at either the north or south pole

Simple test f is structurally unstable if :

- there are nonhiperbolic crit. points on the equator $S^2$
- a trajectory connecting a saddles

Peixoto Theorem gives characterization the structurally stable vector field on a compact two-dim manifold

• They are generic - forms open and dense subsets in $\mathcal{E}$

•• If a vector field $f \in C^1(\mathcal{M})$ is not structurally stable, it belongs ~~to the bifurcation set~~ $C^1(\mathcal{M})$

**Realistic systems should be str. stable**

We always try to convey the features of typical garden variety of dark energy dynam. systems

The exceptional cases not arise very often in applications, they in principle interrupt discussion.

→ This prejudice is shared by all dynamicists (Abraham, Shaw)

In oposite case if additionally the number of critical points and limit cycles is finite $f$ is structurally stable on $S^2$

If the set of all dark energy models with potential $V(x)$ having a certain property such that

- finite number (at finite domain) of limit cycles and single static critical point of saddle type
- there are no trajectories connecting saddles

, then $f \in C^r(M)$, $r \geq 1$ contains an open, dense subset of $C^r(M)$

Corrolary

- only model with single transition from decelerating to accelerating phase is str. stable - generic / Example /

Let $V_1$ & $V_2$ be two dark energy models, then distance between two dark energy models is

$$d(V_1, V_2) = \max\left\{ \sup_{x\in E} \left|\frac{\partial V_1}{\partial x} - \frac{\partial V_2}{\partial x}\right|, \sup_{x\in E} \left|\frac{\partial^2 V_1}{\partial x^2} - \frac{\partial^2 V_2}{\partial x^2}\right| \right\}$$

For example if model (1) & (2) are ΛCDM & PhCDM then d at present epoch is

$$\boxed{d(V_1, V_2) = \frac{3}{2}(1-\Omega_{m,0})|w_x+1|}$$

$p_x = w_x \rho_x$ for phantom $w_x < -1$

Idea of structural stability can be introduced in the language of d metric (Adronov, Pontriagin 1937)

> A vector field f is structurally stable vector field if there is an $\varepsilon > 0$ such that for all $g \in C^1(\mathcal{E})$ with $\|f-g\|_{1} < \varepsilon$ f and g are topologically equivalent

## Structure of ordering $\leq$ in the Ensemble of Dark Energy models

### Degeneracy problem!

$$\boxed{\text{Many models with dramatically different scenarious becomes in good agrement with the present data observations}}$$

This degeneracy can be overcome if we introduce relation "$\leq$" in the class of dark energy models (Ensemble of prototype evolutional scenarious) folowing Akaike and Bayesian informative criterion

From this criterion we can determine (select) the number of essential model parameters providing the preferred fit to the data

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Basing on SNIa data only we find that number of essential parameters is two (M. Szydłowski et al Phys Rev D (2005) M. Szydłowski et al 2005 Phys. Lett B)

$$\boxed{AIC = -2\ln\mathcal{L} + 2d}$$

d – is the number of the model parameter, $\mathcal{L}$ is the maximum likehood

$$\boxed{BIC = -2\ln\mathcal{L} + d\ln N}$$

N is the number of data points used in the fit

- The both criteria preferred fit to the data which minimizes the AIC (or BIC).
- Both have no absolute sense, only relative value has phys: sense. For BIC a difference 2 is regarded as a positive evidence (6 as a strong evidence)

It would be usefull to order dark energy models following AIC & BIC crite.

Table 2

The values of AIC and BIC for distinguished models (Table 1) both for flat and non-flat model.

| case | AIC ($\Omega_{k,0} = 0$) | AIC ($\Omega_{k,0} \neq 0$) | BIC ($\Omega_{k,0} = 0$) | BIC ($\Omega_{k,0} \neq 0$) | |
|---|---|---|---|---|---|
| 0 | 325.5 | 194.4 | 328.6 | 200.5 | |
| 1 | 179.9 | 179.9 | 186.0 | 189.0 | ← ΛCDM |
| 2 | 183.2 | 180.1 | 189.4 | 194.4 | PhCDM $w=-\frac{4}{3}$ |
| 3a | 178.0 | 179.3 | 184.1 | 188.5 | w is fitted |
| 3b | 178.5 | 179.7 | 187.7 | 191.9 | |
| 4a | 181.9 | 181.6 | 191.1 | 193.8 | Bouncing models |
| 4b | 183.9 | 183.6 | 196.2 | 198.8 | |
| 5a | 180.0 | 181.3 | 189.2 | 193.5 | |
| 5b | 182.0 | 183.3 | 194.4 | 198.5 | |
| 6 | 180.3 | 181.9 | 189.4 | 194.1 | |
| 7a | 179.8 | 181.6 | 188.9 | 193.8 | |
| 7b | 180.5 | 182.0 | 192.7 | 197.3 | |

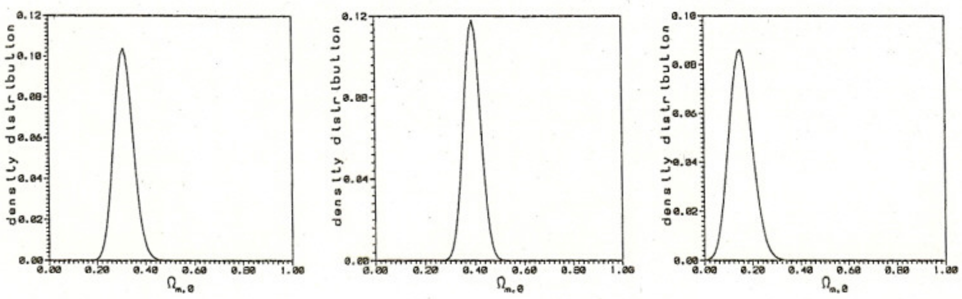
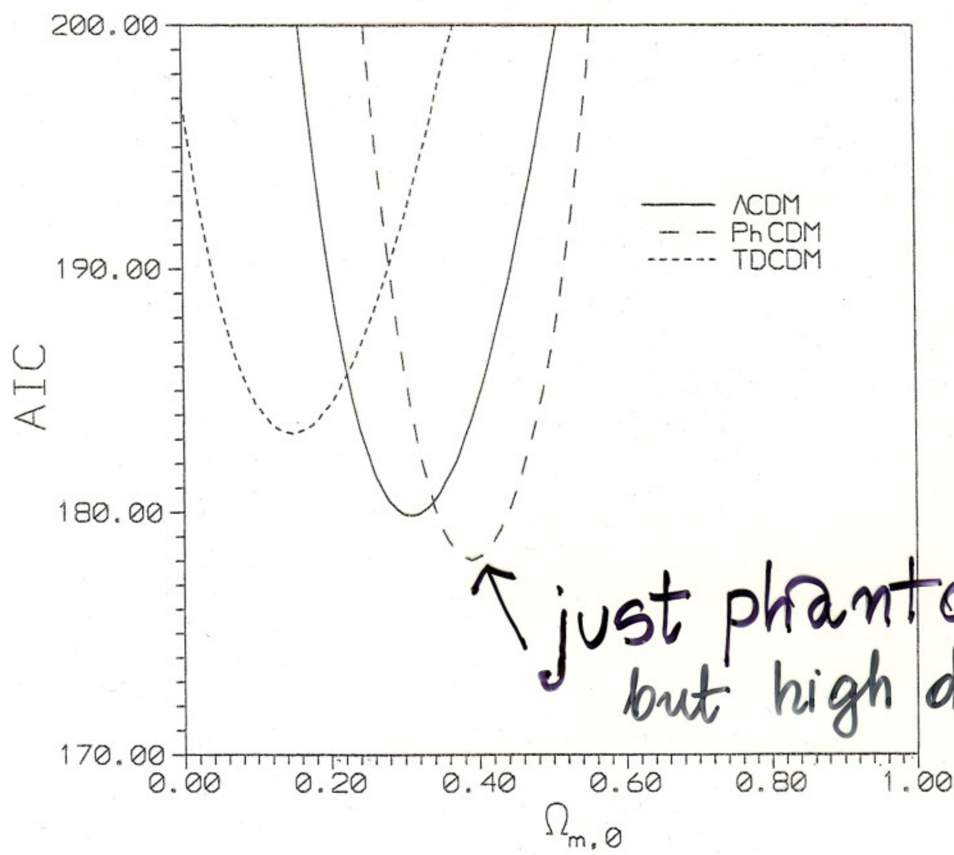



Fig. 1. The one-dimensional probability density distributions (PDFs) for $\Omega_{\mathrm{m},0}$, obtained for (from left to right) the ΛCDM, PhCDM, TDCDM models, respectively.




just phantoms ?
but high density Universe is favored

Fig. 2. The value of AIC in respect to fixed value $\Omega_{\mathrm{m},0}$ for three flat models (with topological defect, with cosmological constant and with phantom) with only one parameter $H_0$ estimated.

Table 2

The values of AIC and BIC for distinguished models (Table 1) both for flat and non-flat model.

| case | AIC ($\Omega_{k,0} = 0$) | AIC ($\Omega_{k,0} \neq 0$) | BIC ($\Omega_{k,0} = 0$) | BIC ($\Omega_{k,0} \neq 0$) |
|---|---|---|---|---|
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| 1 | 179.9 | 179.9 | 186.0 | 189.0 |
| 2 | 183.2 | 180.1 | 189.4 | 194.4 |
| 3a | 178.0 | 179.3 | 184.1 | 188.5 |
| 3b | 178.5 | 179.7 | 187.7 | 191.9 |
| 4a | 181.9 | 181.6 | 191.1 | 193.8 |
| 4b | 183.9 | 183.6 | 196.2 | 198.8 |
| 5a | 180.0 | 181.3 | 189.2 | 193.5 |
| 5b | 182.0 | 183.3 | 194.4 | 198.5 |
| 6 | 180.3 | 181.9 | 189.4 | 194.1 |
| 7a | 179.8 | 181.6 | 188.9 | 193.8 |
| 7b | 180.5 | 182.0 | 192.7 | 197.3 |

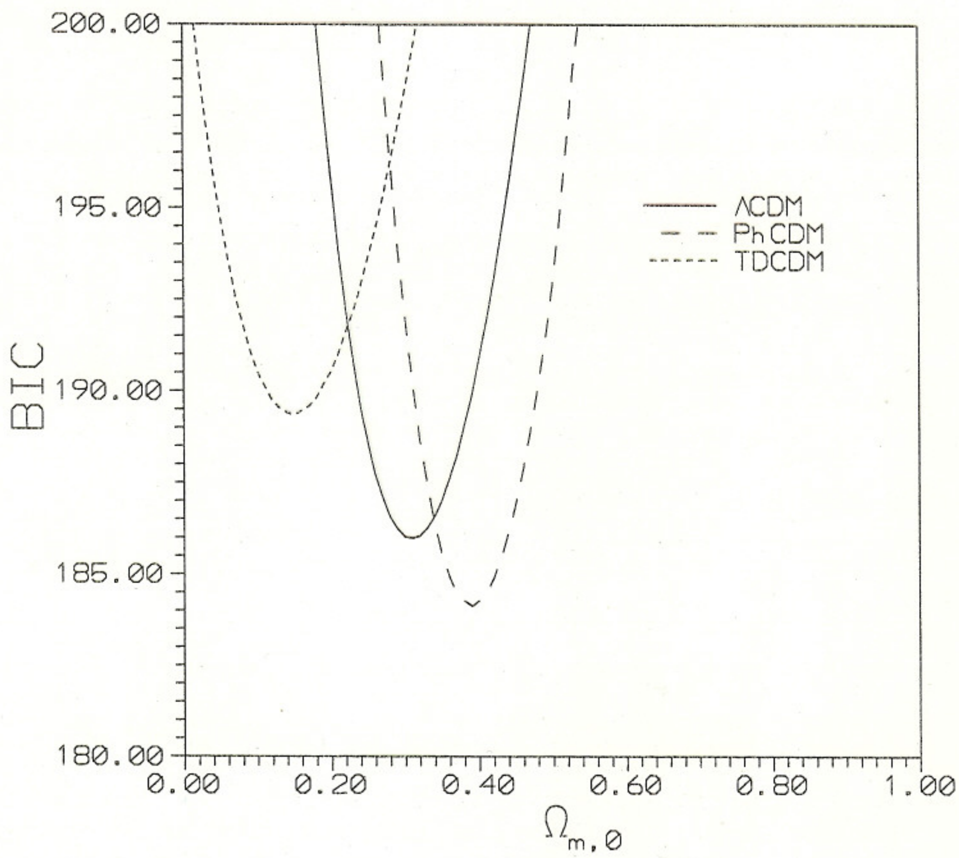



Fig. 3. The value of BIC in respect to fixed value $\Omega_{\mathrm{m},0}$ for three flat models (with topological defect, with the cosmological constant and with phantom) with only one parameter $H_0$ estimated.

Table 3

The values of AIC and BIC for distinguished models (Table 1), with priors $\Omega_{\mathrm{m},0} = 0.3$ both for flat and non-flat model.

| case | AIC ($\Omega_{k,0} = 0$) | AIC ($\Omega_{k,0} \neq 0$) | BIC ($\Omega_{k,0} = 0$) | BIC ($\Omega_{k,0} \neq 0$) | |
|---|---|---|---|---|---|
| 0 | — | 216.9 | — | 220.0 | EdeS |
| 1 | 177.9 | 179.9 | 181.0 | 186.0 | ΛCDM |
| 2 | 190.0 | 178.8 | 193.0 | 184.9 | |
| 3a | 183.9 | 179.6 | 187.0 | 186.7 | $w = -4/3$ fixed |
| 3b | 179.9 | 178.2 | 186.0 | 187.4 | w is fitted |
| 4a | 179.9 | 181.9 | 186.0 | 191.0 | BΛCDM model |
| 4b | 181.9 | 183.9 | 191.1 | 196.1 | |
| 5a | 185.9 | 181.6 | 192.0 | 190.7 | |
| 5b | 187.9 | 183.6 | 197.1 | 195.8 | |
| 6 | 179.5 | 180.1 | 185.6 | 189.3 | |
| 7a | 179.7 | 179.6 | 185.8 | 188.7 | |
| 7b | 179.2 | 180.2 | 188.4 | 192.4 | |

# Conclusions

- singularity can be detected by SNIa data (Riess Gold sample) - good news for Q. Cosm.
- The unified approach to invest. dynamics of cosmological models with dark energy can be given
- We can discuss properties of dynamics in terms of single potential function of the Newtonian type Hamilt. systems

→ SNIa data & Bay. analysis favor FRW models with initial and big rip sing

- We can define in natural way Ensemble of the universes or their evolutional scenarious. From math. point of view it is Banach space which can be useful in methodological analysis (degen. pr)

→ models from which class. C-P emerges from bounce are very special

- Good news is that $\Lambda$CDM (also PhCDM) is structurally stable or property of representing dark energy in term of $\Lambda$ (const) is generic!

$$\mathcal{H} = \frac{p_a^2}{2} - \frac{1}{2}\sum_{i=(m,X)} \Omega_{i,0} f_i(x) \equiv \frac{1}{2}\Omega_{k,0}$$

where

$$\rho_X(a) = \rho_{X,0}\, a^{-3} \exp(-3)\int_1^a \frac{w_X(a)}{a}\, da$$

$$\rho_{eff} = \rho_{m,0}\, a^{-3} + \rho_{X,0} f(a)$$

$$f(1) = 1$$

$$w_{eff} = \frac{p_{eff}}{\rho_{eff}}$$

Let $p_X = w_X(a)\,\rho_X$

$$w_X(a) = \sum_{i=0}^{N} w_i (1-a)^i$$

Then

$$\rho_X = \rho_{X,0}\, a^{(-3)(1+\sum_{i=0}^{N}(-1)^n w_i)} \exp\left\{3\sum_{k=1}^{N}(-1)^k \frac{(1-a)^k}{k} \times \sum_{i=k}^{N}(-1)^{i-k} w_i\right\}$$

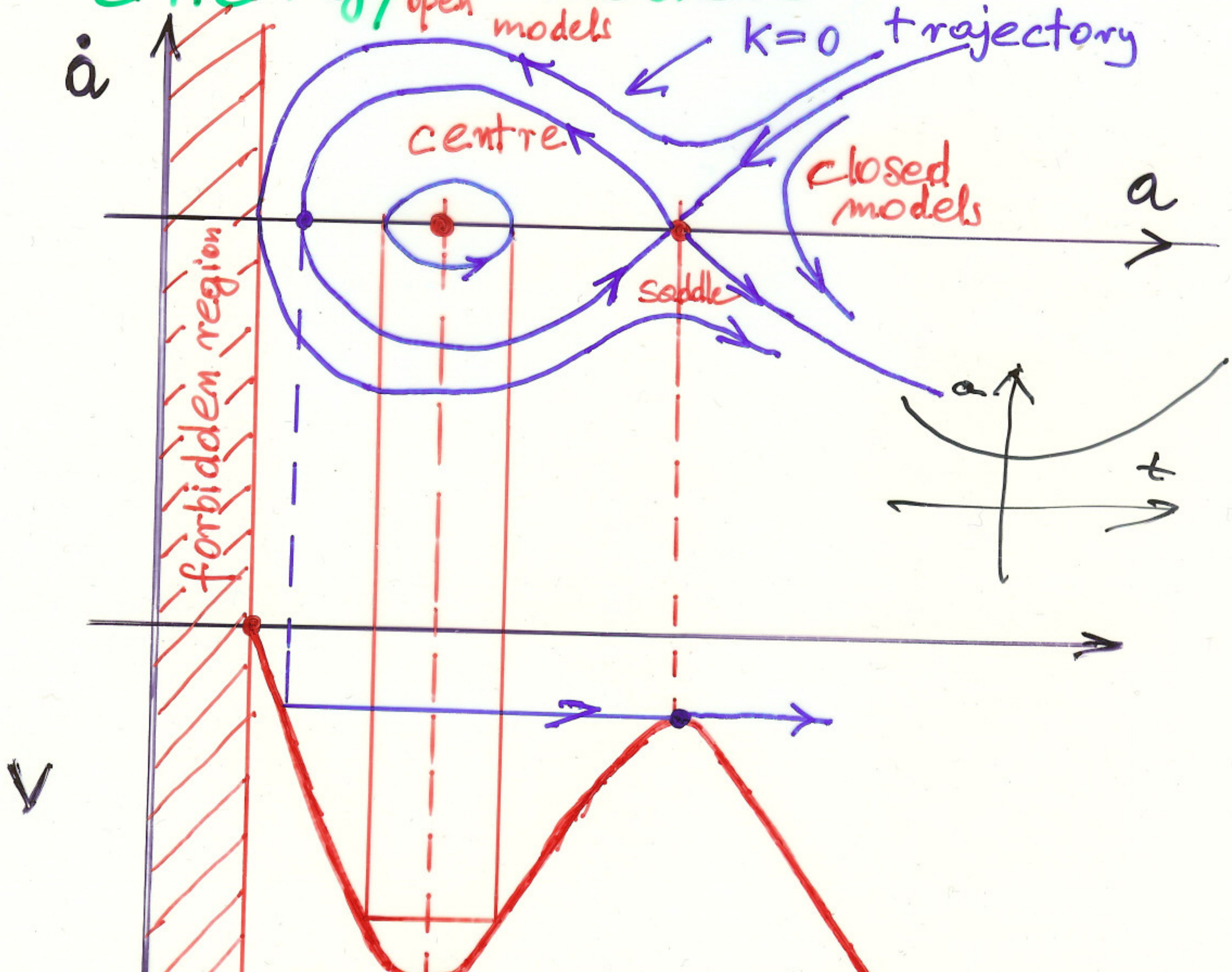
if $N=1$ then

$$\rho_X = \rho_{X,0}\, a^{(-3)(1+w_0-w_1)} \exp\{3w_1(a-1)\}$$

$$\boxed{V(a) = -\frac{1}{2}\Omega_{m,0}\, a^{-1} - \frac{1}{2}\Omega_{X,0}\, a^{-3(1+w_0-w_1)} \times \exp 3w_1(a-1) - \frac{1}{2}\Omega_{k,0}}$$

- $\Lambda$CDM models (as well as PhCDM) forms open and dense subsets in $\mathcal{E}$
- models with bounce and $\Lambda$ are structurally unstable $\Longrightarrow$

bounce is not generic property of the evolutional scenario in the ensemble of dark energy models




Because the presence of centre (non hyp. crit. point) system is struct. unstable