Phenomenological Implications of Discreteness in Loop Quantum Cosmology

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Introduction

LQC - Homogeneous quantization of gravity using techniques of LQG [Bojowald]

Major distinction with standard WdW quantum cosmology - discreteness inherited from LQG, especially in Hamiltonian constraint - discrete difference equation

Phenomenological studies have concentrated on modifications to the inverse volume operator

What are the implications of discreteness:

- description in effective *continuous* picture?
- phenomenological predictions?

Notation

Basic variables:

connection c_I, triad p_I

$\{c_I, p_J\} = \gamma\kappa\delta_{IJ}$

Quantum states $\psi(p_1, p_2, p_3)$

Homogeneous Thiemann constraint $\widehat{C}$

Discrete difference equation:

$$\boxed{\begin{aligned}\widehat{C}\,\psi(p_1,p_2,p_3) = \;& \alpha\,\psi(p_1+\delta p,\, p_2+\delta p,\, p_3) - \beta\,\psi(p_1+\delta p,\, p_2-\delta p,\, p_3)\\ & -\gamma\,\psi(p_1-\delta p,\, p_2+\delta p,\, p_3) + \delta\,\psi(p_1-\delta p,\, p_2-\delta p,\, p_3) + \ldots\end{aligned}}$$

$\delta p \propto \gamma l_p^2 \mu_0$ determines discrete shift in difference equation

Ambiguity parameter $\mu_0 \sim 1$ measures discreteness

μ_0 heuristically related to minimum area eigenvalue of full theory

For simplicity define $\boxed{\epsilon \equiv \mu_0\gamma}$

Compare to standard quantum cosmology - differential equation:

$$\widehat{C}_{WdW}\;\psi(p_1,p_2,p_3) = \sigma\frac{\partial}{\partial p_1}\frac{\partial}{\partial p_2}\;\psi(p_1,p_2,p_3) + \ldots$$

Effective Continuous Framework

Possible to describe discreteness effects in effective *continuous* framework?

Several convergant methods:

- WKB methods [G. Date, G. Hossain, K. Banerjee]
- Path integral considerations [KV]

Both extract an effective continous Hamiltonian constraint

$$\boxed{C_{eff}(p_I, c_I) = \frac{\langle c_I|\widehat{C}|p_I\rangle}{\langle c_I|p_I\rangle}}$$

Validity:

- Hom, isotropic Bianchi I

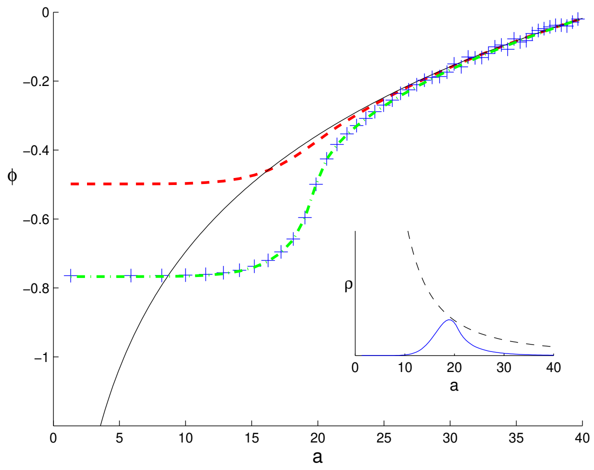
$$\begin{aligned} C_{cl} &= -\frac{3}{\gamma^2\kappa}\sqrt{p}\, c^2 + H_M \\ C_{eff} &= -\frac{3}{\epsilon^2\kappa}\sqrt{p}\, \sin^2(\mu_0 c) + H_M \end{aligned}$$

Evolution of semi-classical states [P. Singh, KV]

green is effective theory

black is classical

blue is semi-classical wave function



Bianchi I Isotropic Model

From effective Hamiltonian constraint derive modified Friedmann equation

$$\boxed{H^2 = \frac{\kappa}{3}\rho_M - \frac{\kappa}{3}\frac{\rho_M^2}{\rho_C}}$$

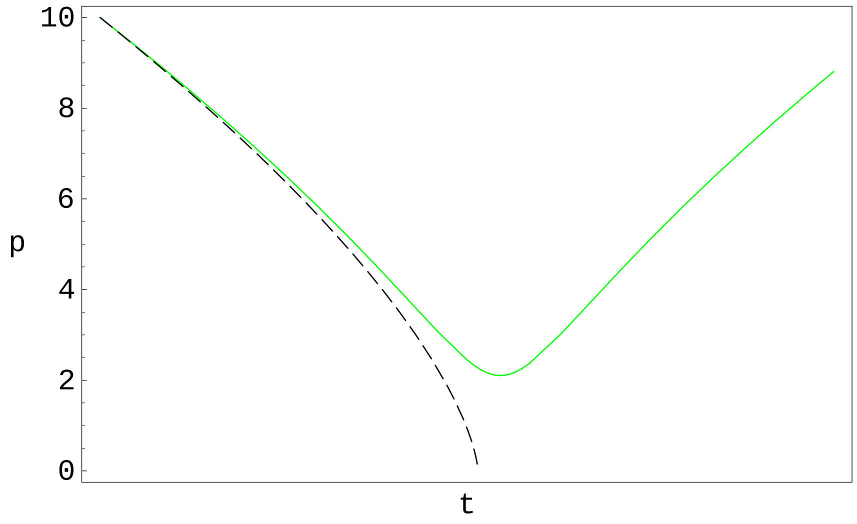
Turn-around (bounce, recollapse) when $\boxed{\rho_M = \rho_c \equiv \frac{3}{\kappa\epsilon^2 a^2}}$

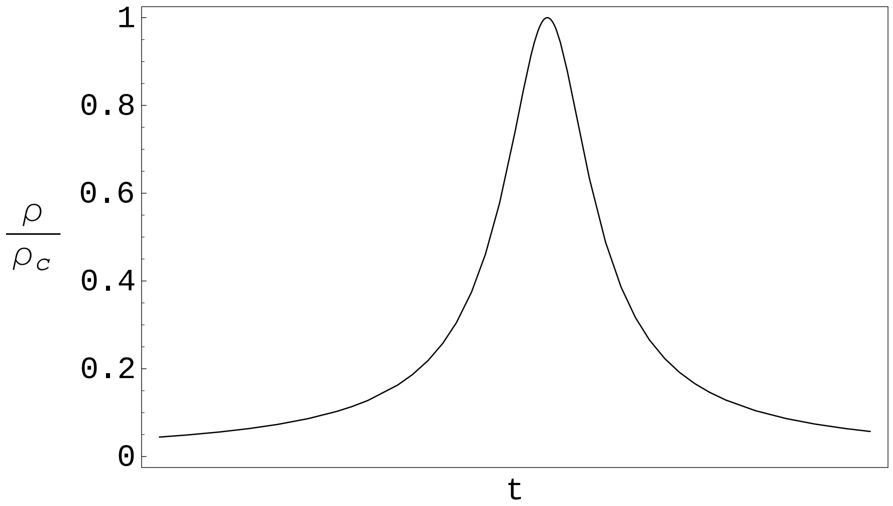
Similar to brane cosmology models with extra time-like dimension

Perfect fluids: $w > -\frac{1}{3}$ bounce

$w < -\frac{1}{3}$ recollapse

Massless scalar field ($w = 1$, see Pawlowski's talk)





Bianchi IX Isotropic Model

$$H^2 = \left(\frac{\kappa}{3}\rho_M - \frac{1}{a^2}\right)\left(1 + \frac{\epsilon^2}{4} - \frac{\kappa\epsilon^2 a^2}{12}\rho_M\right)$$

Two critical densities: $\begin{cases} \rho_{c1} = \frac{3}{\kappa a^2} & \text{(classical turn-around)} \\ \rho_{c2} = \frac{(4+\epsilon^2)}{\epsilon^2}\frac{3}{\kappa a^2} & \text{(quantum, } > \rho_{c1}\text{)} \end{cases}$

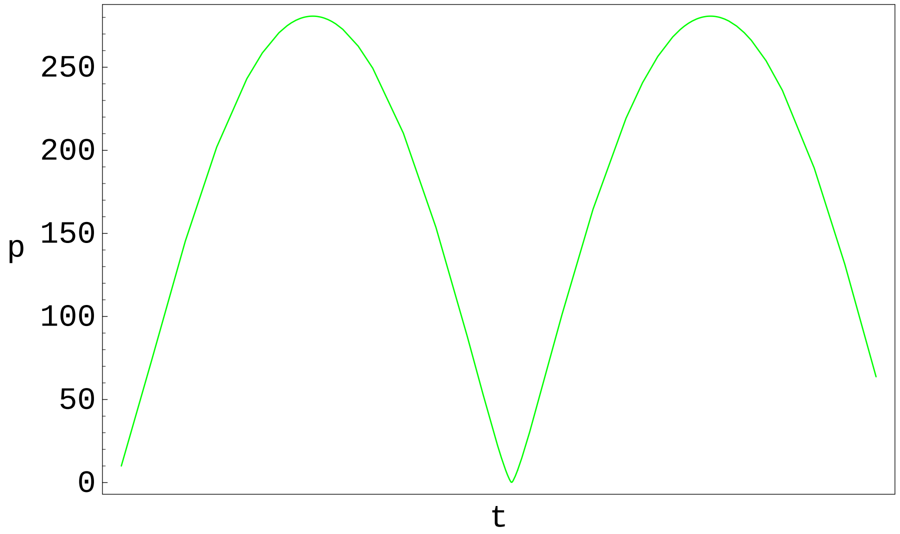
$\rho_M < \rho_{c2}$: running of G and curvature term (relation to RG flow of GR?)

$$H^2 \approx \left(1 + \frac{\epsilon^2}{4}\right)\left(\frac{\kappa}{3}\rho_M - \frac{1}{a^2}\right)$$

Cyclic style models

Scalar field with just inverse volume corrections [J. Lidsey, D. Mulryne, N. Nunes, R. Tavakol]

Discrete corrections give cycles of bouncing/recollapsing with more generic matter:



Bianchi I Anisotropic Model

Classically anisotropies typically dominate over matter at small volumes

$$H^2 = \frac{\kappa}{3}\rho_M + \frac{\Sigma^2}{a^6}$$

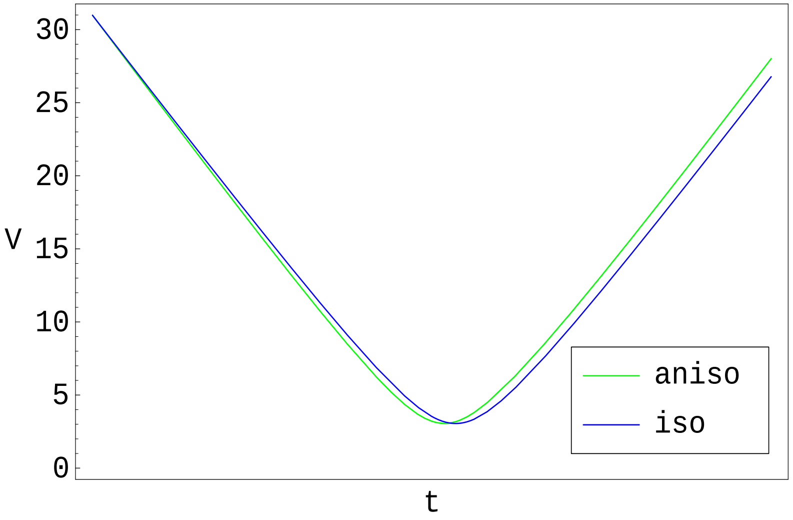
Do bounces still occur? Do anisotropies blow up at small volume?

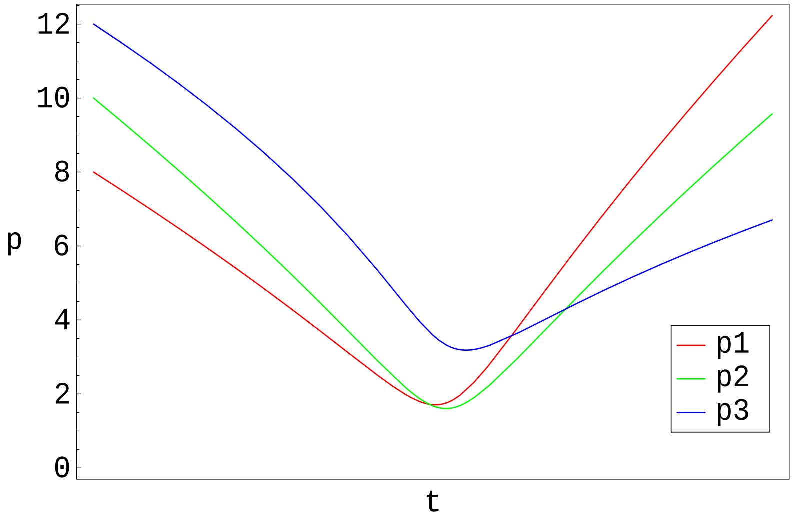
$$\boxed{C_{eff} = -\frac{1}{\kappa}\left[\sqrt{\frac{p_2 p_3}{p_1}}\frac{\sin(\mu_0 c_2)\sin(\mu_0 c_3)}{\epsilon^2} + cyclic\right] + H_M}$$

Bounce occurs without matter [G. Date]

Massless scalar field:

$$\boxed{H^2 = \frac{\kappa}{3}\rho_M + \frac{\Sigma^2}{a^6} - \epsilon^2\frac{\alpha_1+\alpha_2+\alpha_3}{18\,a^{12}}\left[a_1^2\alpha_1^2(\alpha_2+\alpha_3) + cyclic\right] + \mathcal{O}(\epsilon^3)}$$





Effective theory issues

Problems with effective theory:

isotropic flat model $\rho_c = \frac{3}{\kappa\epsilon^2 a^2}$ - what is a?

$\rho_c \sim \frac{1}{a^2}$ decreases for large volume - sub Planckian universe can still have QG effects

example - cosmological constant model recollapses at large volume ($\rho_\Lambda = const$), recollapse during standard inflation

Another symptom:

Discrete volume step of difference equation grows: $V(p + \delta p) - V(p) \propto \mu_0 \sqrt{p}$

Directly related with $\rho_c \sim \frac{1}{a^2}$

Growth of the discrete volume steps unexpected from full theory considerations: Heuristically thus expect μ_0 to *decrease* with p

Consistency grounds of the quantization scheme indicates the $\mu_0 \propto \frac{\Delta l_p}{\sqrt{p}}$ with Δ depending on area gap of LQG.

Critical density now constant and on order of Planckian $\rho_c = \frac{3}{\kappa\gamma^2\Delta^2 l_p^2}$

Qualitative picture remains the same - bounces still present - quantitative nature changes

Running of G and curvature in Bianchi IX model

Outlook

Can understand singularity resolution in LQC in new semi-classical light: approach to singularity replaced with bounce even in spatially flat models

Can effective picture be extended to less symmetric models?

- Inhomogeneous spherically symmetric models
- Collapse - bounce also?

Essential to probe inhomogeneous models

- Need closer connection to full theory - range of validity of LQC
- Does picture change? Still bounces? Inhomogeneities blow up?
- Perturbations in bouncing universe (essential to construct viable cyclic models)