From semi-classical LQC to Friedmann Universe

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Modified cosmological equations within semi-classical LQC

Modified Friedmann's equation:

Singh, Vandersloot, [gr-qc/0507029]

$$\boxed{H^2 = \frac{\kappa}{3}\tilde{\rho}\left(1 - \frac{\tilde{\rho}}{\rho_c}\right)}, \quad \tilde{\rho} = \frac{1}{2}\frac{\psi^2}{D} + V(\phi), \quad H = \frac{d\ln a}{dt}, \quad \kappa = 8\pi G,$$

$$\rho_c = \frac{3}{\kappa(\mu_0\gamma a)^2}, \quad \begin{array}{l}\mu_0 = \sqrt{3}/4, \\ \gamma = 0.2375\ldots,\end{array}$$

Modified Klein-Gordon equation:

$$\boxed{\dot{\psi} = -3H\psi\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right) - D\frac{\partial V}{\partial\phi},} \qquad \psi = \dot{\phi} \equiv \frac{d\phi}{dt},$$

$$D(q) = \left(\frac{8}{77}\right)^6 q^{3/2}\left\{7\left[(q+1)^{11/4} - |q-1|^{11/4}\right] - 11q\left[(q+1)^{7/4} - \mathrm{sign}(q-1)|q-1|^{7/4}\right]\right\}^6,$$

$$q = \left(\frac{a}{a_*}\right)^2, \qquad a_* = \left(\frac{\kappa\gamma\mu_0}{3}J\right)^{1/2}.$$

Domain of validity

Vandersloot, [gr-qc/0502082]

These equations are valid if $\mu_0 c \ll 1,$

In the classical limit $c = \gamma \dot{a},$

So that we have: $\mu_0 \gamma \dot{a} \ll 1,$

This is still valid for large volumes!

$$\mu \gg 2J\mu_0.$$

Reduction to the form of Friedmann's equations

Modified cosmological equations can be written in the Friedmann's form:

$$H^2 = \frac{\kappa}{3}\rho_{eff}, \qquad \frac{d\rho_{eff}}{dt} + 3H(\rho_{eff} + p_{eff}) = 0,$$

with the following notation:

$$\rho_{eff} = \tilde{\rho}\left(1 - \frac{\tilde{\rho}}{\rho_c}\right), \qquad \left\{\begin{array}{l} \tilde{\rho} = \dfrac{1}{2}\dfrac{\psi^2}{D} + V(\phi), \\ \\ \tilde{p} = \dfrac{1}{2}\dfrac{\psi^2}{D}\left(1 - \dfrac{1}{3}\dfrac{d\ln D}{d\ln a}\right) - V. \end{array}\right.$$

$$p_{eff} = \tilde{p} - \frac{\tilde{\rho}}{\rho_c}(\tilde{\rho} + 2\tilde{p}),$$

Lidsey, e.a., PRD70(2004)063521

Some implications of modified Friedmann's equations

1. Constraints on the energy density:

$$H^2 = \frac{\kappa}{3}\tilde{\rho}\left(1 - \frac{\tilde{\rho}}{\rho_c}\right) \longrightarrow \boxed{\tilde{\rho} \leq \rho_c}$$

2. Solution for the energy density:

$$0 \leq \rho_- \leq \frac{\rho_c}{2}$$

$$\frac{\rho_c}{2} \leq \rho_+ \leq \rho_c,$$

$$\rho_\pm = \frac{\rho_c}{2}\left(1 \pm \sqrt{1 - \frac{12H^2}{\kappa\rho_c}}\right) \longrightarrow \boxed{-\sqrt{\frac{\kappa\rho_c}{12}} \leq H \leq \sqrt{\frac{\kappa\rho_c}{12}}}$$

3. Hubble parameter equation:

$$\dot{H} = -\kappa\frac{\psi^2}{2}\left(1 - \frac{2\tilde{\rho}}{\rho_c}\right)\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a}\right) + \frac{\kappa}{6H}\dot{\rho}_c\left(\frac{\tilde{\rho}}{\rho_c}\right)$$

Effective theories within s.c. LQC

$$
\begin{cases}
H^2 = \dfrac{\kappa}{3}\tilde{\rho}\cancel{\left(1 - \dfrac{\tilde{\rho}}{\rho_c}\right)}, \\[2ex]
\dot{H} = -\kappa\dfrac{\psi^2}{2}\cancel{\left(1 - \dfrac{2\tilde{\rho}}{\rho_c}\right)}\cancel{\left(1 - \dfrac{1}{6}\dfrac{d\ln D}{d\ln a}\right)} + \cancel{\dfrac{\kappa}{6H}\rho_c\left(\dfrac{\tilde{\rho}}{\rho_c}\right)} \\[2ex]
\dot{\psi} = 3H\psi\cancel{\left(1 - \dfrac{1}{3}\dfrac{d\ln D}{d\ln a}\right)} + D\dfrac{\partial V}{\partial \phi},
\end{cases}
$$

$$a \ll a_*, \quad \rho \simeq \rho_c$$

(ET-II)

$$a \ll a_*, \quad \rho \ll \rho_c$$

(ET-I)

$$a \gg a_*, \quad \rho \simeq \rho_c$$

(ET-III)

Classical limit: $a \gg a_*$, $\rho \ll \rho_c$

Massless scalar field within ET-I

Vereshchagin, JCAP, 07 (2004) 013

Equations:

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_P^2}\left[\frac{\psi^2}{2D} + V\right],$$

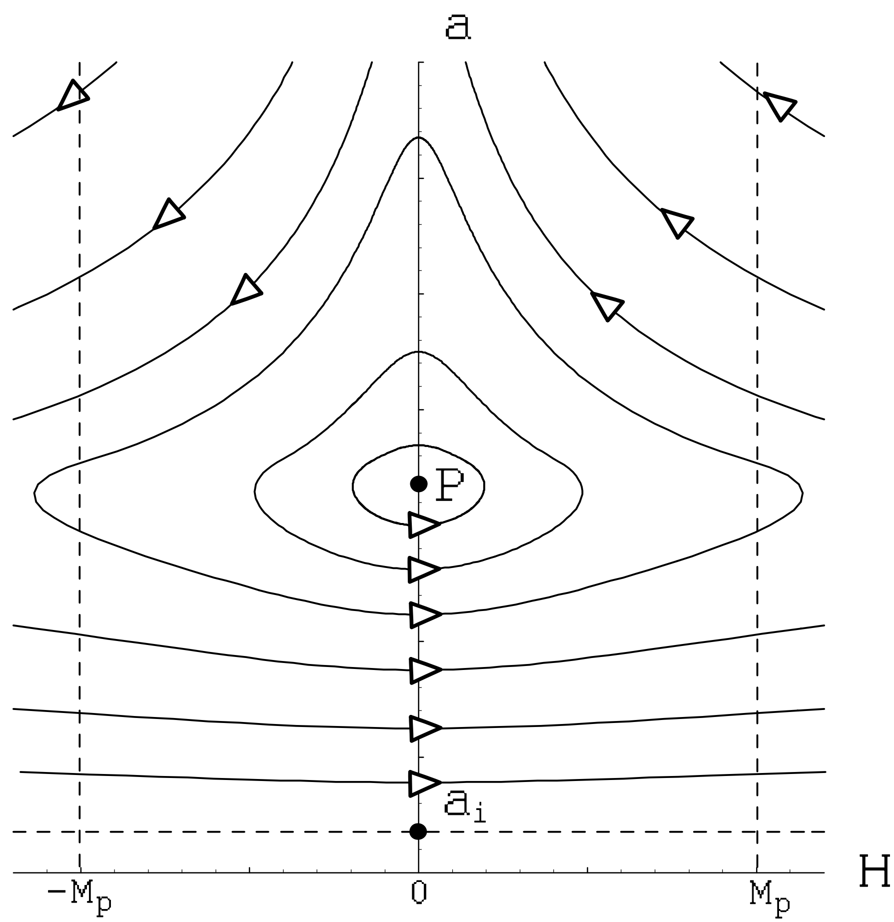
$$\dot{\psi} = -3H\psi + \frac{\dot{D}}{D}\psi - D\frac{\partial V}{\partial \varphi},$$

$$\dot{H} + H^2 = -\frac{8\pi}{3M_P^2}\frac{\psi^2}{D}\left[1 - \frac{\dot{D}}{4HD}\right] + \frac{8\pi V}{3M_P^2},$$

For V=0 the phase space is reduced to 2D one {H,a}

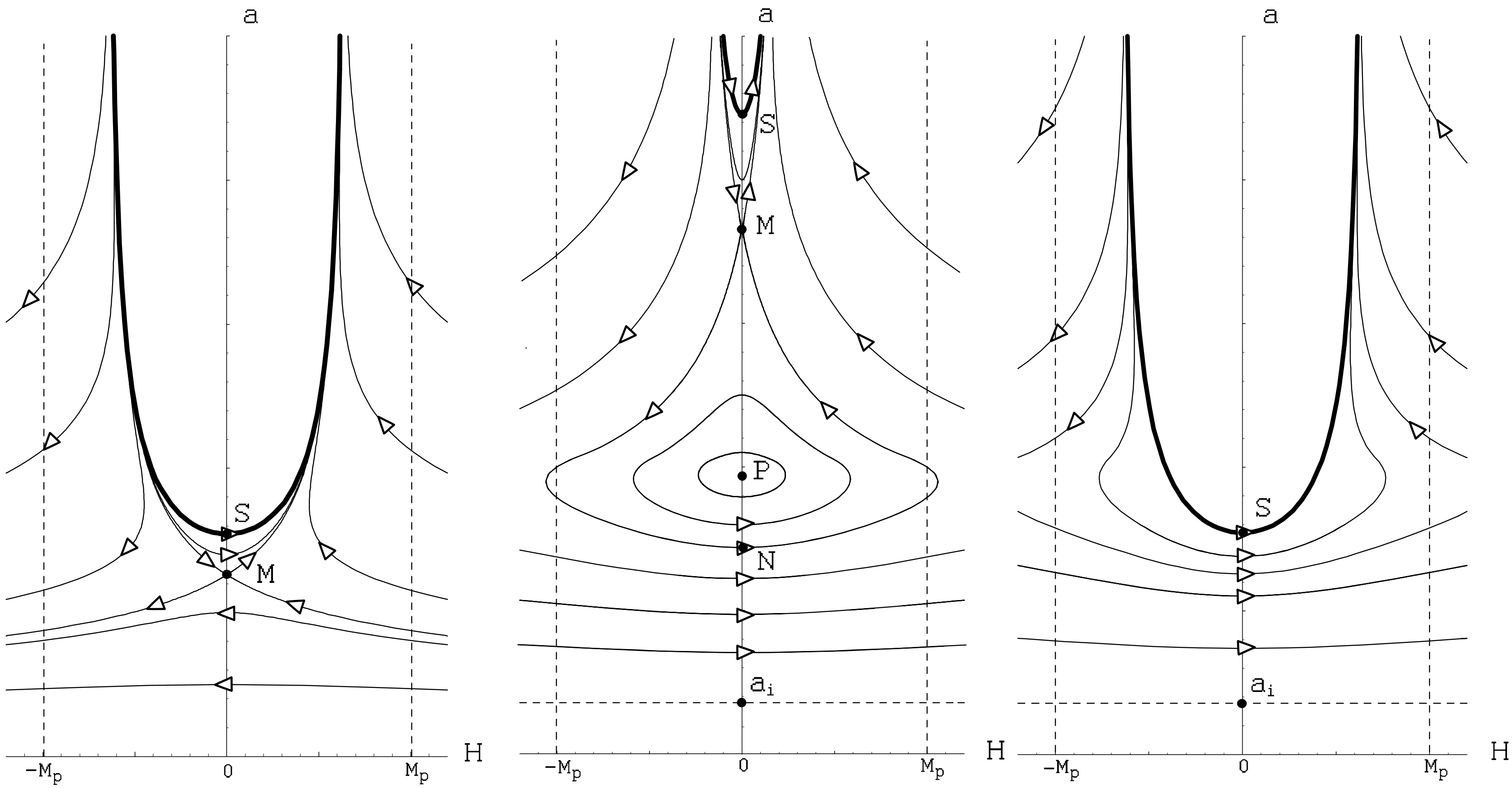
There is only one singular point: center P.

Singularity is absent for all phase trajectories within bounds – $M_p<H<M_p$



Massive scalar field within ET-I

Vereshchagin, JCAP, 07 (2004) 013



General relativity

LQC: $V<V_c$

$V>V_c$

M – saddle, P – center, S – limiting value of the scale factor.

Singularity avoidance for k=0

$$H^2 = \frac{\kappa}{3}\tilde{\rho}\left(1 - \frac{\tilde{\rho}}{\rho_c}\right), \quad H(0) = 0 \quad \longrightarrow \quad \rho(0) = \rho_c,$$

otherwise

$$\phi(0) = \dot{\phi}(0) = 0$$

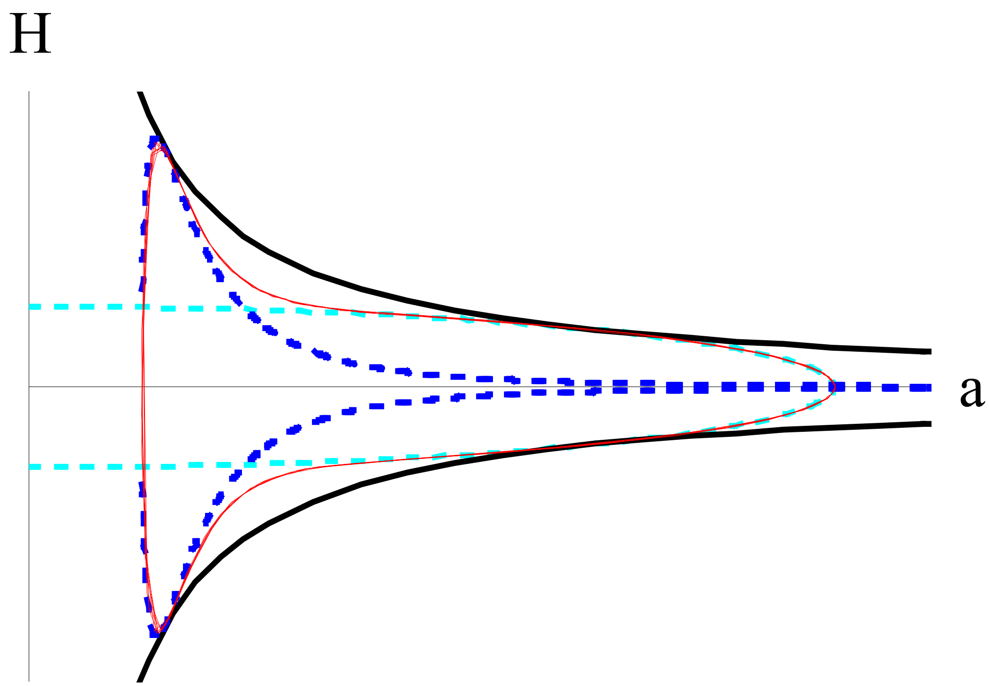
$$\dot{H} = -\kappa\frac{\psi^2}{2}\left(1 - \frac{2\tilde{\rho}}{\rho_c}\right)\cancel{\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a}\right)} - \frac{\kappa a^2}{\alpha}\tilde{\rho}^2$$

ET-III:

$$\begin{cases} H^2 = \frac{\kappa}{3}\left(\frac{1}{2}\psi^2 + V\right)\left[1 - \frac{a^2}{\alpha}\left(\frac{1}{2}\psi^2 + V\right)\right], \\ \dot{H} = -\kappa\frac{\psi^2}{2}\left[1 - \frac{2a^2}{\alpha}\left(\frac{1}{2}\psi^2 + V\right)\right] - \frac{\kappa a^2}{\alpha}\left(\frac{1}{2}\psi^2 + V\right)^2, \\ \dot{\psi} + 3H\psi + D\frac{\partial V}{\partial \phi} = 0. \end{cases}$$

Cycles within ET-III

- Maximum expansion point is ***potential-dominated***
- Bounce is ***kinetic-dominated***
- The presence of boundaries is essential



Blue – kinetic-dominated solution,

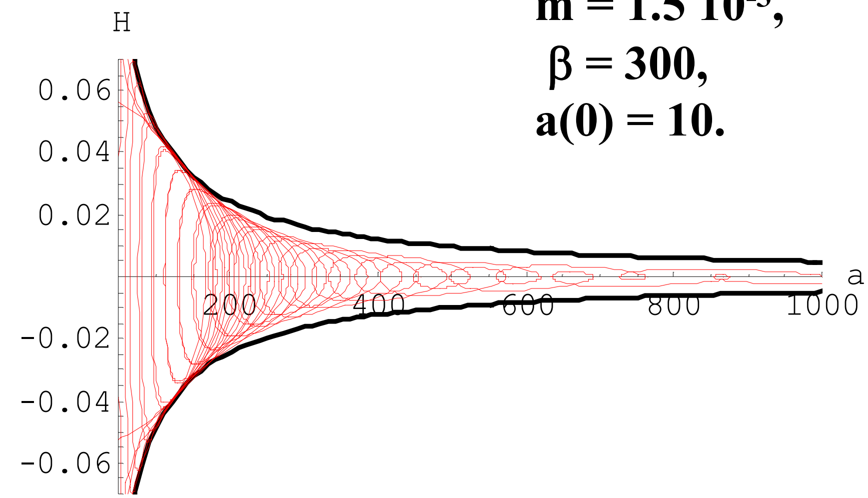
Azure – potential-dominated solution,

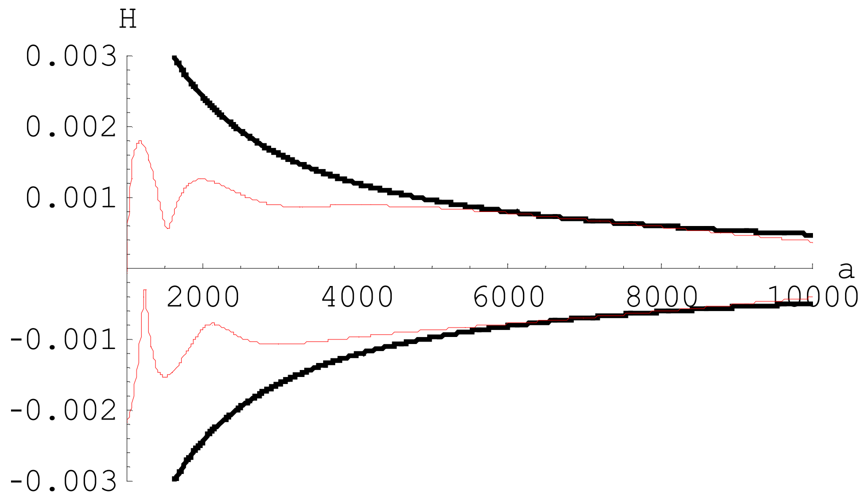
Red – numerical solution,

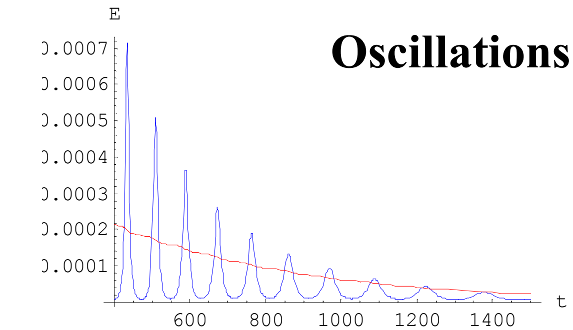
Black - boundaries

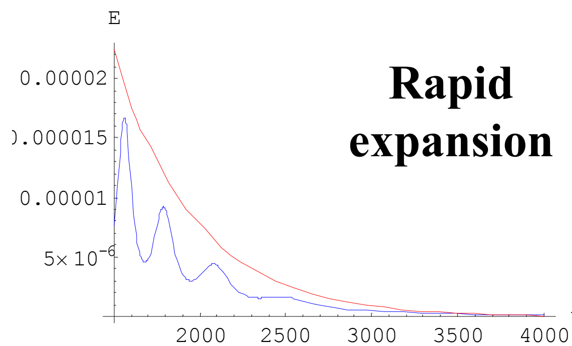
Example

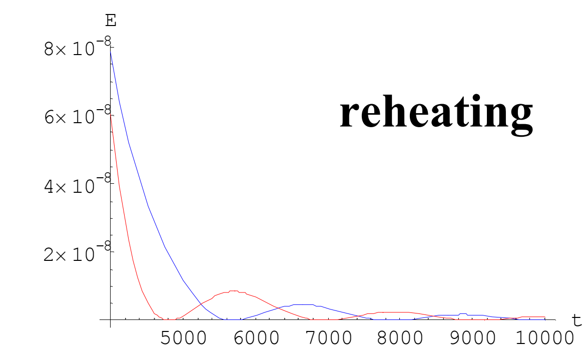
m = 1.5 10^{-3},
β = 300,
a(0) = 10.











Size of the Universe?...

$$\rho_c = \frac{\alpha}{a^2}, \ \alpha = \frac{3}{\kappa(\mu_0\gamma)^2} \approx 11.28 G^{-1},$$

$$Ha \leq \sqrt{\frac{\kappa\alpha}{12}} \approx 5 \quad \Rightarrow \quad a_{today} \leq 5 H_0^{-1}$$

... is of the order of $O(H^{-1})$?!

Validity: $Ha \ll (\mu_0\gamma)^{-1} \approx 10.$

Cosmological eras

- **Boundary:** $H \propto a^{-1}$,
- Oscillations: $H \propto a^{-4}$,
- "Potential-dominated": $H \approx \text{const}$,
- Reheating: $H \propto a^{-3/2}$,
- Radiation-dominated: $H \propto a^{-2}$,
- Matter-dominated: $H \propto a^{-3/2}$,
- Acceleration: $H \approx \text{const}$.

Conclusions and perspectives

- Significance of quantum corrections for the scale factor and/or energy density ➔ effective theories;
- ET-III outcomes: absence of singularity, size of the Universe, initial conditions?

- Present day acceleration…close to maximal expansion?
- What about k=1 models?
- What is the picture for ET-II?