Quantum Black Holes I

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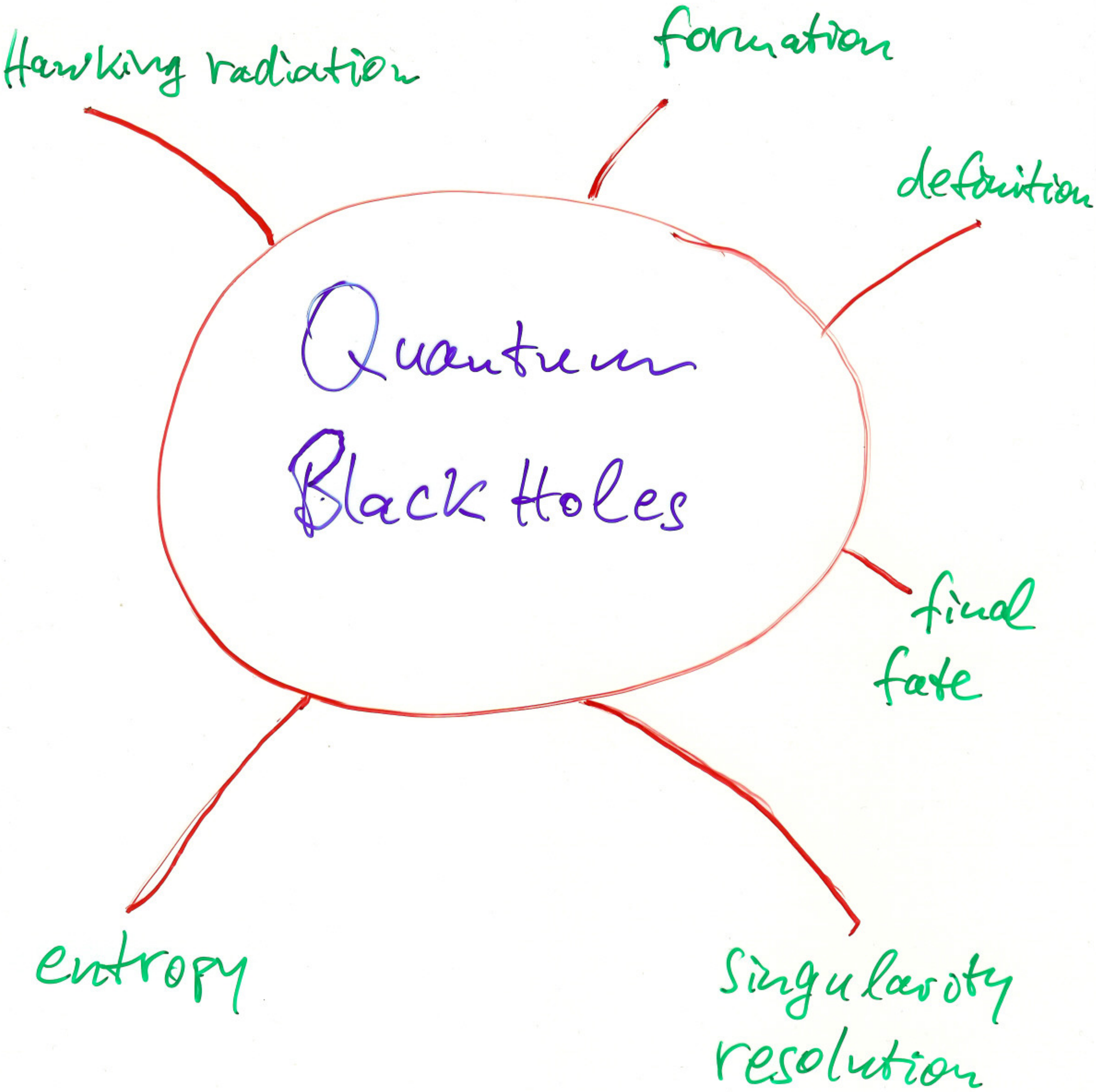

Quantum Black Holes
Hawking radiation
formation
definition
final fate
singularity resolution
entropy

Standard model

spherically symmetric gravity
coupled to matter (scalar field usually)

$$ds_\Sigma^2 = \Lambda^2(r,t)\,dR^2 + R^2(r,t)\,d\Omega$$

Λ, R, scalar fields Φ

$\rightarrow$ 2D field theory midisuperspace

Classically

System has sectors with BH formation
and sectors w/o bhs (cf. Choptuik)

Task

Quantize the system
Come up with good def. of Quantum BH
Investigate collapse problem
Evaporation?

⋮

70s BCNM
Unruh

fully gauge fixed $\rightarrow$ Φ only d.o.f.

$$H_{red} = \int dr \left(\frac{P_\phi^2}{4r^2} + r^2 \phi'^2 \right) \exp\left(\int_\infty^r S_\phi(r')\, dr' \right)$$

completely untractable

90s Kastrup, Thiemann
Kuchař

fully constrained system
difficult to make contact with
standard semicl. calculations like
Hawking radiation

$\Downarrow$

our strategy: partial gauge fixing $\Lambda = 1$

R, Φ remaining d.o.f.

Classical Setting

$$q_{ab} = n_a n_b + \frac{R^2}{r^2}(e_{ab} - n_a n_b)$$

$$\tilde{\pi}^{ab} = \frac{P_\Lambda}{2} n^a n^b + \frac{r^2 P_R}{4R}(e^{ab} - n^a n^b)$$

(R, P_R) (Φ, P_Φ) phase space var.

fall off conditions chosen by demanding Painleve - Gullstrand form of metric in fully gauge fixed form

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}}\,dt\right)^2 + r^2 d\Omega^2$$

regular at $r = 2M$, covers entire spatial hypersurface

$$R = r + O(r^{-1/2-\varepsilon}) \qquad P_R = A\frac{r^{-1/2}}{2} + O(r^{-1-\varepsilon})$$

$$\phi = Br^{-1/2} + O(r^{-3/2-\varepsilon}) \qquad P_\phi = Cr^{1/2} + O(r^{-\varepsilon})$$

Quantization

Methods inspired by LQG

Ex point particle

(x,p) $\quad H = \frac{p^2}{2m} + V(x)$

basis states $\quad |e^{ixp}\rangle \equiv |x\rangle$

$\langle x | y \rangle = \delta_{x,y}$ Kronecker delta

→ plane waves normalizable
(necessary for singularity resolution)

$\hat{x}|x\rangle = x|x\rangle$

$\hat{p}$ does not exist "non-regular" repr.

only $\hat{u}_a = \widehat{e^{iap}}$

$\hat{u}_a|x\rangle = |x-a\rangle$

$$\hat{H}_a = \frac{1}{2m} \frac{2 - \hat{u}_a - \hat{u}_{-a}}{a^2} + \hat{V}(x)$$

Scalar field = "one particle at each point $x \in \mathbb{R}_0^+$"

basis states

$$|e^{i\sum a_k P_R(x_k)}, e^{i\sum b_l P_\Lambda(x_l)}, e^{i\sum c_m P_\phi(x_m)}\rangle$$

$$\equiv |a_1 \ldots a_{N_1}, b_1 \ldots b_{N_2}, c_1 \ldots c_{N_3}\rangle$$

$$\langle a_1 \ldots a_{N_1}, b_1 \ldots b_{N_2}, c_1 \ldots c_{N_3} | a'_1 \ldots a'_{N_1}, b'_1 \ldots b'_{N_2}, c'_1 \ldots c'_{N_3}\rangle$$

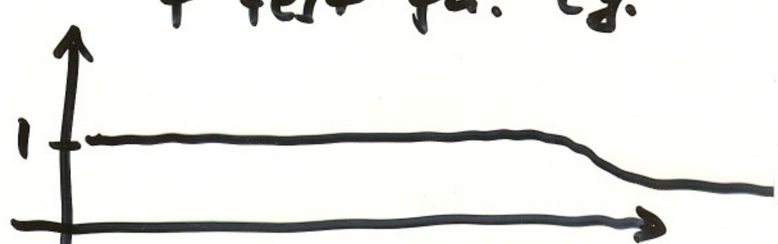
$$= \delta_{a_1 a'_1} \cdots \delta_{c_{N_3} c'_{N_3}}$$

- basis states like mode states in Fock space, but in x-space
- represent excitations of quantum field at locations $\{x_k\}$

basic operators

$$R_f = \int_0^\infty dr\, R(r) f(r)$$

f test fn. eg.



$$\hat{R}_f |a_1 \ldots a_N\rangle = 2 l_P^2 \sum_k a_k f(x_k) |a_1 \ldots a_N\rangle$$

$$\widehat{e^{i\lambda_j P_R(x_j)}} |a_1 \ldots a_N\rangle = |a_1, \ldots a_j - \lambda_j, \ldots a_N\rangle$$

creates excitation λ_j at location x_j

Singularity Resolution

The "building block", contained in quantities of interest, that diverges at classical singularity is $\frac{1}{R}$

ex. $\tilde{\Pi} = \frac{1}{2}\left(\frac{P_\Lambda}{R^2} + \frac{P_R}{R}\right)$

H_{red} reduced Hamiltonian

H Hamiltonian constraint

→ strategy: construct $\widehat{\frac{1}{R}}$ operator which has finite eigenvalue at class. singularity

$$\frac{1}{|R_f|} = \left(A\, e^{-iP_R}\left\{\sqrt{|R_f|},\, e^{iP_R}\right\}\right)^2$$ Thiemann trick

⇒ $\widehat{\frac{1}{|R_f|}}$ by "hat operation"

Eigenvalue of $\widehat{\frac{1}{R}}$ at class. singularity

$\frac{z}{l_p{}^2}$ large but finite!

$\rightarrow$ Kinematical resolution of class. singularity

also dynamics remains finite at quantum level

What is a Quantum Black Hole?

BHs are intimately connected to the notion of horizon

Many types of horizon:

Event, apparent, trapping isolated, ...

Which one to take?

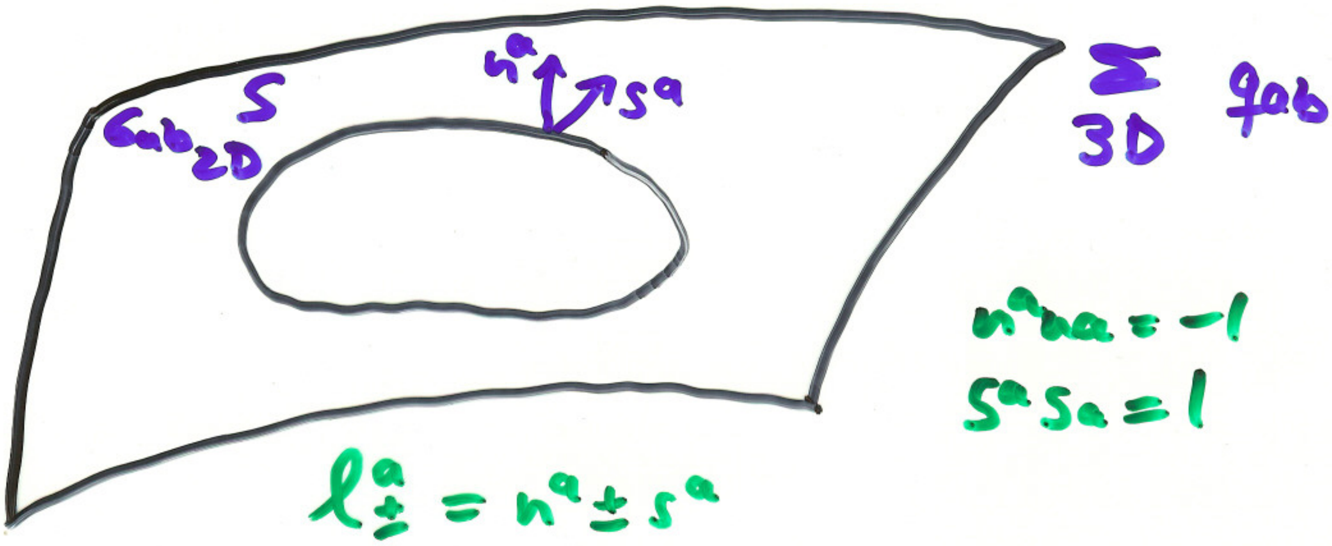
Want s.th. physically intuitive connected to trapping of light
practical, i.e. useful for "real life" e.g. numerical relativity
genuinely quantum, i.e. no fixing of classical boundaries

→ trapping horizons

Definition of Quantum Black Holes

Idea

Define BHs via trapped surfaces (Hayward)



$n^a n_a = -1$

$s^a s_a = 1$

$l_\pm^a = n^a \pm s^a$

$\theta_\pm = \sigma^{ab} D_a l_{\pm b}$

expansion of $\frac{\text{out}}{\text{in}}$ going light rays

S is outer trapped if $\theta_+ \leq 0 \quad \theta_- < 0$

marginally outer trapped (MOTS) $\quad \theta_+ = 0 \quad \theta_- < 0$

In this framework:

$$\theta_\pm = -\frac{1}{2}\left(4RR' \pm P_\Lambda\right)$$

Quantization:

$\hat{R}$

$\hat{P}_\Lambda$ (see talk by Vigar)

$\hat{R}'$ finite difference of $\hat{R}$ operators

$\Rightarrow$ $\hat{\theta}_\pm$ horizon operator

tool to probe whether state $|\Psi\rangle$ describes QBH or not:

if there is $\bar{r}$ such that

$$\langle\Psi|\hat{\theta}_+(\bar{r})|\Psi\rangle = 0$$

and $\langle\Psi|\hat{\theta}_-(\bar{r})|\Psi\rangle < 0$

then $|\Psi\rangle$ describes quantum spacetime with a qbh with horizon at $\bar{r}$

Horizon size: $R_H = \langle\Psi|\hat{R}|\Psi\rangle$

Conclusion

- Have kinematical quantum theory
- singularity resolution
- genuinely quantum definition of a black hole

Next step:

Implement dynamics

$\hookrightarrow$ next talk