Loop quantization as a continuum limit

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ABOUT LOOP QUANTIZATION

$$\begin{array}{c} \text{Classical Theory} \\ \text{kinematics, dynamics} \end{array} \xrightarrow{\text{LQ}} \begin{array}{c} \text{Quantum Theory} \\ \textbf{kinematics}, \textit{dynamics} \end{array}$$

Types of theories:

- background metric independent **or** background metric dependent
- gauge **or** sigma modles **or** scalar field

ABOUT THIS WORK

- Contributes to *dynamical* aspects of **LQ**
- Applies to all types of theories
- Brings all of LGT (metric dependent) to LQ

ORGANIZATION

1. General framework

2. Explicit example: 2d Ising field theory
   (see Elisa Manrique's poster for details)

3. Discussion

1. GENERAL FRAMEWORK

We work on the Euclidean description of QFT
(Direct Hamiltonian treatment also possible)

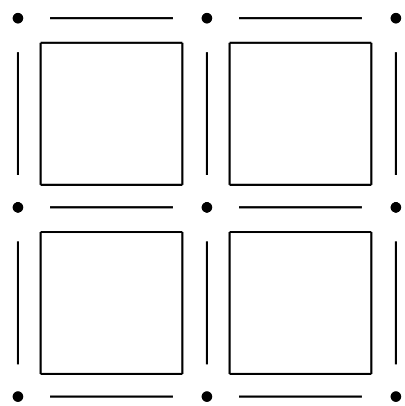
Implementation of Wilson's RG

Effective theos (scale C) Fundamental theo (M)

$$\{(\mathcal{A}_C, \mu_C)\}_{C \text{celldec}(M)} \longrightarrow (\bar{\mathcal{A}}_M, \mu_M)$$

Example of cellular decomposition:

$\mathbb{R}^2 = \cup_\alpha c_\alpha, \qquad c_\alpha \cap c_\beta =$ if $\alpha \neq \beta$



Structure: Directed partial order ($\leq$)
Consequence: the limit $C \to M$ makes sense (and if it exists it is unique)

Example of continuum limit:
f : Cell decs$(M) \to \mathbb{R}$

$$\lim_{C \to M} f(C) = f(M)$$

means that

$$\forall \quad \epsilon \geq 0 \quad \exists C_0 :$$
$$|f(C) - f(M)| \leq \epsilon \quad \forall C \geq C_0$$

$\bar{\mathcal{A}}_M$ space of "loopy" Euclidean histories (G-sigma models here)

$s \in \bar{\mathcal{A}}_M \quad \Longleftrightarrow \quad s(p) \in G$ for all $p \in M$

$\mathcal{A}_C$ space of effective Euclidean histories (C-constant histories)

$s \in \mathcal{A}_C \subset \bar{\mathcal{A}}_M \quad \Longleftrightarrow$
$s(p) = s(q) \Leftarrow$ there is $c_\alpha \in C$ with $p, q \in c_\alpha$

Inclusion $\mathcal{A}_C \to \bar{\mathcal{A}}_M$

Meaning of the continuum limit

$$\lim_{C \to M} (\mathcal{A}_C, \mu_C) = (\bar{\mathcal{A}}_M, \mu_M)$$

means that

- EventuallyIn($\mathcal{A}_C$) is dense in $\bar{\mathcal{A}}_M$

- Given any $f \in$ Cyl($\bar{\mathcal{A}}_M$) the continuum limit measure acts as

$$\mu_M(f) \doteq \lim_{C \to M} \mu_C(i_C^\star f)$$

Notice that the construction holds also for (smaller) more economic families of cell decs.; e.g. regular cell decs.

Constructing the measures μ_C: renormalization prescriptions

$$C_1 \qquad\qquad \leq \qquad\qquad C_2$$

$$(\mathcal{A}_{C_1}, \mu_{C_1}) \underset{\pi_{C_2 C_1}}{\overset{i_{C_1 C_2}}{\rightleftarrows}} (\mathcal{A}_{C_2}, \mu_{C_2})$$

($\pi_{C_2 C_1}$ is a coarse graining map. See Elisa's poster for an explanation)

Since they are two effective descriptions of the same system

$$\pi_{C_2 C_1 \star} \mu_{C_2} \approx \mu_{C_1}$$

- Strong (stupid) renorm. presc.
  (All C_1 obs. equally measured at C_2)

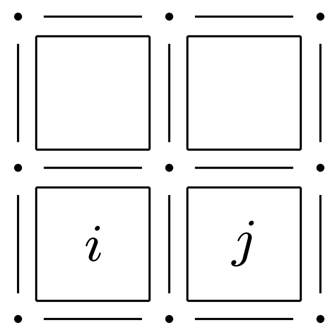
- Weak (inteligent) renorm. presc.
  (Some C_1 obs. equally measured at C_2)

If $\mu_{C_i} = \mu(\beta_{C_i}) \quad \Rightarrow \quad \beta_{C_{i+1}} = \beta_{C_{i+1}}(\beta_{C_i})$
If iterable $\Rightarrow$ RG flow (direction $C \to M$)

Can the approx. $\pi_{C_{i+1}C_{i}\star}\mu_{C_{i+1}} \approx \mu_{C_i}$ improve as $C_i \to M$?

Yes, if there is critical phenomena at the fixed point $\lim_{C\to M} \beta_C = \beta_M = \beta_{critical}$

2. EXPLICIT EXAMPLE: 2D ISING QFT



$\mu_{\beta_C} \qquad \longleftrightarrow \qquad n$-point functions

$$\mu_{\beta_C}(s_{p_1} \cdot \ldots \cdot s_{p_n}) = \frac{1}{Z_C} \sum_s \frac{s_{p_1} \ldots s_{p_n}}{M(\beta_C)^n} \exp\left[-\beta_C \sum_{(ij)} s_i s_j\right]$$

Theorem 1 (McCoy, Tracy, Wu) *Choose* β_C *as to maintain the physical correlation length constant. Then*

$$\lim_{C \to \mathbb{R}^2} \mu_{\beta_C}(s_{p_1} \cdot \ldots \cdot s_{p_n}) = \mu_{\mathbb{R}^2}(s_{p_1} \cdot \ldots \cdot s_{p_n})$$

The limit exists and it is explicitly given in terms of special functions (Elisa's poster).

Thus, we have constructed a non trivial Euclidean QFT on $\bar{\mathcal{A}}_M$.

3. DISCUSSION

1. Introduced background structure. What about symmetries (rotational if metric, diff if metric independent)?

2. What about a Hamiltonian (real time) formulation?

3. What about importing numerical work from LGT?

4. Can we apply this form of LQ to General Relativity?