Taming the cosmological constant in 2D causal quantum gravity with topology change

Talk prepared for the

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Potsdam, Oct. 2005

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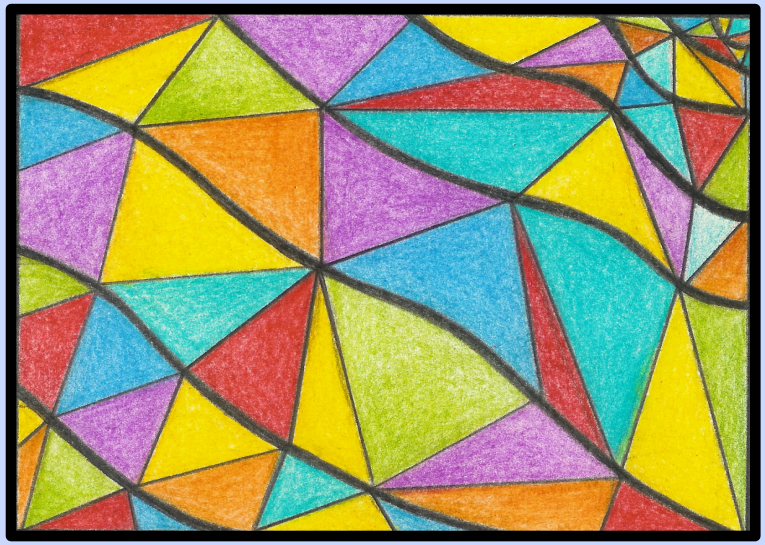

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related work:
[hep-th/0507012]
R. Loll, W. Westra, S. Z.

Empty space and the need for quantum gravity

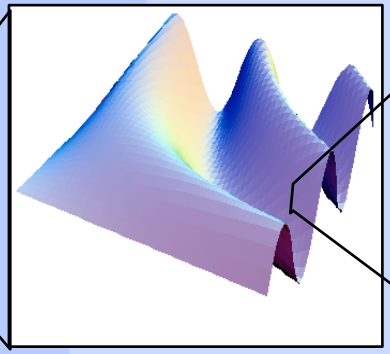
Quantum mechanics:
Heisenberg uncertainty relation:

$$(\Delta\text{Distance})(\Delta\text{Energy}) \geqslant \hbar$$

General relativity:
Einstein's equivalence principle
Gravity couples to all forms of matter and energy.

Energy deforms space-time

smooth, flat



smooth, curved

Fluctuating geometry?

Fluctuating topology?

non-trivial micro-structure

Need for theory of quantum geometry $\cong$ quantum gravity

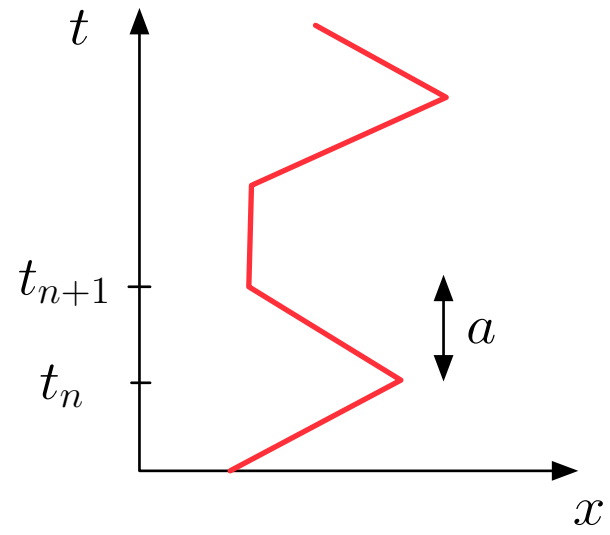
Path integral formulation: Quantum mechanics

Quantum particle:

$$\text{propagator}(x_1, x_2, \Delta t) = \sum_{\substack{\text{all particle path } p \\ x_1 \longrightarrow x_2}} e^{iS^{\text{particle}}[p]}$$

classical action

- propagating "virtual" particle
- piecewise straight path



Continuum limit $\rightsquigarrow$ quantum mechanics

$$\xrightarrow{a \longrightarrow 0} \int \mathcal{D}x e^{iS[x]}$$

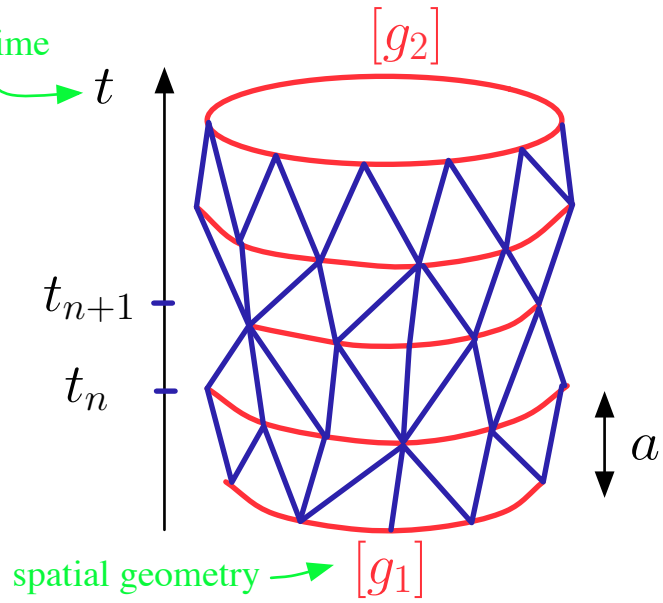
Path integral formulation: Dynamical triangulations (DT)

Quantum geometry:

$$\text{propagator}([g_1],[g_2],\Delta t) = \sum_{\substack{\text{all space-time geometries} \\ [g_1] \to [g_2]}} e^{iS^{\text{Einstein}}[g]}$$

metrics modulo diffeomorphism

- piecewise straight geometries (analogue QM particle)
- flat building blocks $\rightsquigarrow$ in 2D triangles



Continuum limit $\rightsquigarrow$ quantum gravity

$$\xrightarrow{a \to 0} \int \mathcal{D}[g_{\mu\nu}] e^{iS^{\text{Einstein}}[g_{\mu\nu}]}$$

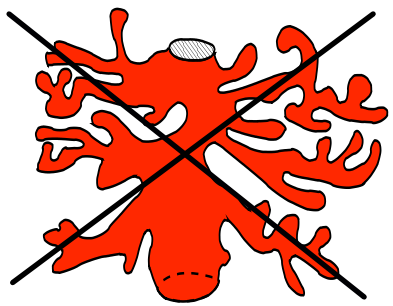
From dynamical triangulations to causal dynamical triangulations

Features of DT:
- nonperturbative
- background independent
- no new symmetries
  (unlike string theory)

Problems:
need convergence of PI $\rightsquigarrow$ Wick rotation

Way around:
Euclidean formulation (DT)
treats time like all other coordinates
$\rightsquigarrow$ no light-cones, no causality

New idea: Causal dynamical triangulations (CDT):
- take Lorentzian structure seriously
- Minkowskian building blocks
- exists well-defined Wick rotation



Recent results in d=4:
- PI well behaved
- at large scales dynamically generated 4-dim. universe [hep-th/0411152]
- at short scales dynamical reduction to two dimensions [hep-th/0505113]

CDT in two dimensions: Analytically solvable

First derived in
J. Ambjørn, R. Loll
[hep-th/9805108]

Einstein's equations are classically trivial:

$$S[g] = \Lambda \int_M d^2x\sqrt{|g|} - 2\pi K\chi(M)$$

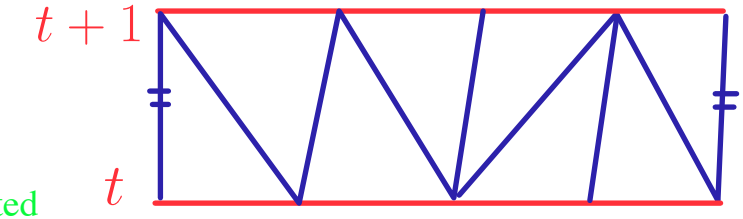
$\chi(M) = 2 - 2\mathrm{g}$

Transfer matrix:

$$< l_2|\hat{T}|l_1 >= \sum_{\substack{\text{all triangulations } T \\ l_1 \to l_2}} e^{-\lambda N(T)}$$

bare coupling

already Wick rotated



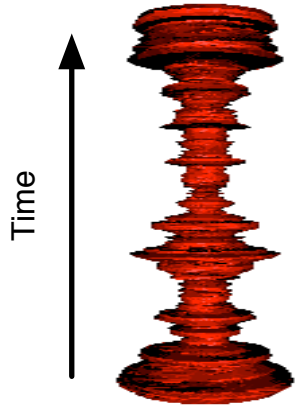
Continuum limit $a \to 0$**:**

- Renormalization: $\psi(L)$, Λ
- effective quantum Hamiltonian

$$\hat{H}(L, \frac{\partial}{\partial L}) = -L\frac{\partial^2}{\partial L^2} - \frac{\partial}{\partial L} + 2\,\Lambda\, L.$$

Fluctuating "quantum universe"
with average spatial length

$$\langle L \rangle \sim \frac{1}{\sqrt{\Lambda}}$$



$d_H = 2$

Distinct from Euclidean QG

CDT with topology change

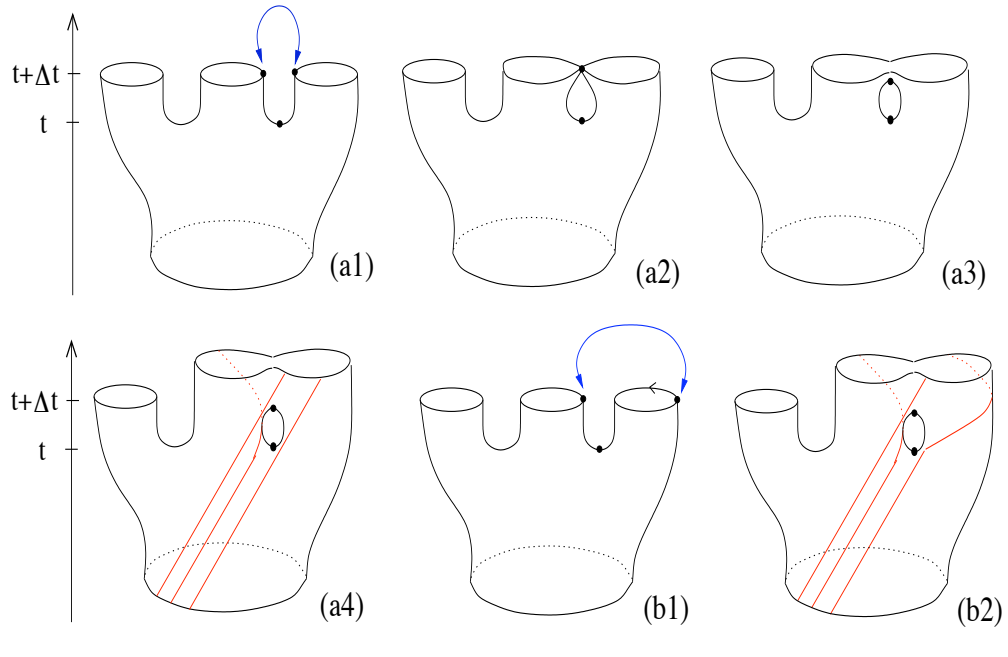
R. Loll, W. Westra
[hep-th/0306183]
R.Loll, W. Westra, S.Z.
[hep-th/0507012]

Aim: Include the sum over topologies in the PI

$$Z(\Lambda, G_N) = \sum_{topol.} \int \mathcal{D}[g_{\mu\nu}]\, e^{iS[g_{\mu\nu}]}$$

Problem:
super-exponential growth in the number of configurations

Solution:
Use causal structure to exclude "bad" topology changes (which lead to macroscopic causality violations) from the PI.



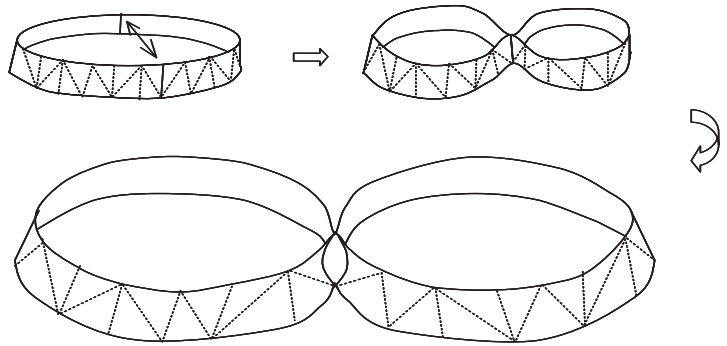
Only allow for "untwisted" wormholes of infinitesimal duration.

Implementation of wormholes

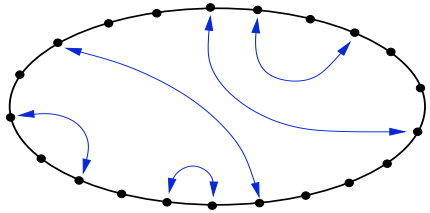
Construction in framework of CDT

wormholes of duration $\Delta t = 1$

- identify two time-like edges
- cut open the geometry along this line



Apply this procedure g times to construct g wormholes (genus g surface)



- no intersecting arches

Discrete Solution: Transfer matrix:

bare couplings

$$< l_2|\hat{T}|l_1 >= \sum_{g \geqslant 0}^{[N/2]} \sum_{\substack{\text{all T for fixed g:} \\ l_1 \to l_2}} e^{-\lambda N - 2\kappa g}$$

Continuum limit: Find suitable double scaling limit for λ and κ

Continuum and double scaling limit

Scaling relations:

$$x = 1 - aX + \mathcal{O}(a^2)$$
$$y = 1 - aY + \mathcal{O}(a^2)$$
$$g = \frac{1}{2}(1 - a^2\,\Lambda) + \mathcal{O}(a^3)$$
$$h^2 = \frac{1}{2}h_{ren}^2\,\Lambda\,(X+Y)\,a^3, \quad h_{ren} = e^{-2\pi/G_N}$$

Loop-loop correlator:

$$G_{\Lambda,G_N}(L_1,L_2;T) = \frac{\omega}{\sqrt{L_1 L_2}}\frac{e^{-\omega(L_1+L_2)\coth(\omega T)}}{\sinh(\omega T)} I_1\left(\frac{2\omega\sqrt{L_1 L_2}}{\sinh(\omega T)}\right), \quad \omega = \sqrt{2\Lambda(1-h_{ren}^2)}$$

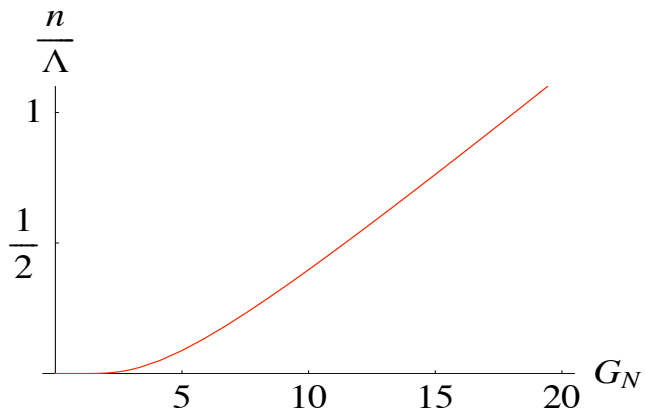
Calculate further dynamical quantities and observables.

Observables and how to tame the cosmological constant

Fluctuating universe
with finite density of wormholes

$$n = \frac{1}{e^{\frac{4\pi}{G_N}} - 1}\Lambda$$

$\eta = \dfrac{n}{\Lambda}$ is relative scale between topological and cosmological fluctuations.

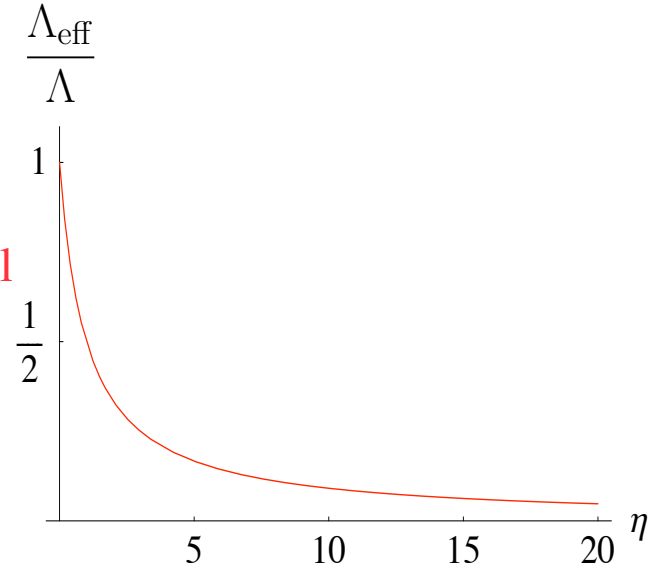


Effective quantum Hamiltonian:

$$\hat{H}(L, \frac{\partial}{\partial L}) = -L\frac{\partial^2}{\partial L^2} - \frac{\partial}{\partial L} + 2\frac{1}{1+\eta}\Lambda L$$

effective cosmological constant Λ_{eff}

Presence of wormholes is accompanied by a decrease in the effective cosmological constant.
(cf. Coleman's mechanism)



Summary and Outlook:

- **CDT leads to a well defined continuum theory of quantum gravity in 2D**

- **Therefore no need for a fundamental discrete length scale!**

- **Sum over topologies can explicitly be performed in the background independent framework of CDT**

- **Presence of wormholes is accompanied by a decrease in the effective cosmological constant**

Causal dynamical triangulations recently gave interesting results in d=4
It is a promising candidate for a theory of quantum gravity